PMS 547 Foundations of Databases
Homework II
Due on May 11, 2010.

1. Let $\mathcal{I}$ be an an arbitrary interpretation. Prove or disprove (by giving a counterexample) that the following subsumption statements of $\mathcal{ALC}$ (where $\mathtt{A}$ and $\mathtt{B}$ are concepts and $\mathtt{R}$ a role) are satisfied by $\mathcal{I}$:

   $$\forall \mathtt{R}.(\mathtt{A} \sqcap \mathtt{B}) \sqsubseteq \forall \mathtt{R}.\mathtt{A} \sqcap \forall \mathtt{R}.\mathtt{B}, \quad \exists \mathtt{R}.\mathtt{A} \sqcap \exists \mathtt{R}.\mathtt{B} \sqsubseteq \exists \mathtt{R}.(\mathtt{A} \sqcap \mathtt{B})$$

   Your proofs must use the semantics of $\mathcal{ALC}$.

   (10+10 marks)

2. Consider the following English sentences:

   - Yannis is a person.
   - The only kind of coffee that Yannis drinks is frappe.
   - A Greek is a person who drinks only frappe coffee.[1]
   - Yannis is Greek.

   Now answer the following questions:

   (a) Give an $\mathcal{ALC}$ knowledge base KB which formalizes the first three of the above sentences and an $\mathcal{ALC}$ formula ϕ that formalizes the fourth sentence.

   (b) Construct an interpretation $\mathcal{I}$ which satisfies KB.

   (c) Construct another interpretation $\mathcal{I}_1$ which does not satisfy KB.

   (d) Use tableau techniques to prove that $KB \models \phi$. Your proofs in this and following exercises must be very detailed (in particular, formulas participating in a proof must be numbered so that the application of each rule clearly shows what formulas are involved).

   (10+5+5+10=30 marks)

3. Consider the following English sentences (a more involved version of the previous example):

   - Yannis is a person.
   - Frappe is a kind of coffee. Nescafe frappe is a kind of frappe.
   - The only kind of coffee that Yannis drinks is Nescafe frappe.
   - A Greek is a person who drinks only Frappe coffee.
   - Yannis is Greek.

   Now answer the following questions:

   (a) Give an $\mathcal{ALC}$ knowledge base KB which formalizes the first four of the above sentences and an $\mathcal{ALC}$ formula ϕ that formalizes the fifth sentence.

[1] I admit that this is a rather short-sighted definition!

(b) Use tableau techniques to prove that $KB \models \phi$.

This proof involves dealing with arbitrary TBoxes, not just TBoxes with acyclic terminologies that we covered in the lectures. There is an extra transformation rule that you will need in this case. The details are given in the paper "Franz Baader. Description Logics. In Reasoning Web: Semantic Technologies for Information Systems, 5th International Summer School 2009, volume 5689 of Lecture Notes in Computer Science, pages 1-39. Springer-Verlag, 2009." which is available from `http://lat.inf.tu-dresden.de/research/papers.html`. This is a nice recent survey of description logics, so, if I were you, I would read it very carefully! See also the relevant part of the description logic slides by Renate Schmidt available at `http://www.cs.manchester.ac.uk/~schmidt/COMP6016/2009-2010/dlWeek2.pdf`.

(10+30=40 marks)

4. Consider the following English sentences:

- Walt is a person.
- Walt has three distinct pets: Huey, Dewey and Louie.
- Huey, Dewey and Louie are animals.
- An animal lover is a person which has at least three pets that are animals.
- Walt is an animal lover.

Now answer the following questions:

(a) Give an $\mathcal{ALCQ}$ knowledge base KB which formalizes the first four of the above sentences and an $\mathcal{ALCQ}$ formula ϕ that formalizes the fifth sentence.

(b) Use tableau techniques to prove that $KB \models \phi$.

The description logic $\mathcal{ALCQ}$ is an extension of $\mathcal{ALC}$ with qualified number restrictions. $\mathcal{ALCQ}$ and tableau proof techniques for it are also covered in the paper by Franz Baader cited above.

(10+30=40 marks)