Managing Uncertainty and Vagueness in Semantic Web

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Abstract. Semantic Web has been designed for processing tasks without human intervention. In this context, the term machine processable information has been introduced. In most Semantic Web tasks, we come across information incompleteness issues, aka uncertainty and vagueness. For this reason, a method that represents uncertainty and vagueness under a common framework has to be defined. Semantic Web technologies are defined through a Semantic Web Stack and are based on a clear formal foundation. Therefore, any representation scheme should be aligned with these technologies and be formally defined. As the concept of ontologies is significant in the Semantic Web for representing knowledge, any framework is desirable to be built upon it. In our work, we have defined an approach for representing uncertainty and vagueness under a common framework. Uncertainty is represented through Dempster-Shafer model, whereas vagueness has been represented through Fuzzy Logic and Fuzzy Sets. For this reason, we have defined our theoretical framework, aimed at a combination of the classical crisp DL $\mathcal{ALC}$ with a Dempster-Shafer module. As a next step, we added fuzziness to this model. Throughout our work, we have implemented metaontologies in order to represent uncertain and vague concepts and, next, we have tested our methodology in real-world applications.

Keywords: Uncertainty · Vagueness · Dempster-Shafer Model · Description Logics · Semantic Web

1 Introduction

The Internet has paved the way for the evolution of alternative methods of communication. E-commerce, e-banking and online stores are some of them. Traditionally, computers were designed for performing numerical calculations. In addition, the content of Web information has been designed for human consumption, i.e. it is *human oriented*. The evolution of search engines gave a boost at the popularity of WWW, but at the same time made it necessary for the existence of a Web (or Web information) suitable for machines (or agents).

Towards this concept, Semantic Web was the vision of Tim Berners-Lee who stated: "*Machines become capable of analyzing all the data on the Web - the*

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content, links, and transactions between people and computers. "A Semantic Web", which should make this possible, has yet to emerge, but when it does, the day-to-day mechanisms of trade, bureaucracy and our daily lives will be handled by machines talking to machines, leaving humans to provide the inspiration and intuition. The "intelligent agents" people have touted for ages will finally materialize" [3].

The Semantic Web will contribute in the evolution of many web applications [1], such as Knowledge Management, Business-to-Computer, Electronic Commerce and Wikis.

Reliability, ambiguity or incompleteness issues are usual problems considering Web information, resulting in deficient knowledge. Any method that represents machine-oriented information should provide a well-defined description of imprecise knowledge [23].

Imprecise knowledge is usually divided into:

– *Uncertainty*
– *Vagueness*

Uncertainty refers to situations of information incompleteness whereas vagueness describes imprecise information, i.e concepts with not well-defined meaning. Generally, uncertainty and vagueness are considered two different notions and as such different theories have been defined for representing them. Probability theory, Dempster-Shafer theory and Possibility theory are some frameworks designed for uncertainty representation [6]. On the other hand, Fuzzy Logic and Fuzzy Sets [34] is the theory that lies behind vagueness representation. In many cases, we come across situations where both uncertainty and vagueness coexist. Thus, we need a common framework in order to represent uncertainty and vagueness concepts. Both notions can be defined as *imperfect information*.

Regarding Semantic Web, *ontologies* is the core concept for knowledge representation. Ontologies are represented through the Web Ontology Language (OWL) with OWL2 being the current version [33]. Description Logics (DLs) [2] have been employed extensively in Semantic Web, as they are the logics behind the most widely used version of OWL, OWL-DL. The necessity to capture uncertain and vague knowledge in Semantic Web has been employed in extensions of DLs, resulting in Probabilistic [21], Possibilistic [26] and Fuzzy extensions [29, 32]. These extensions capture the problem of uncertainty and vagueness separately and not as a common framework.

2 Objectives

2.1 Main Idea

The main goal of this dissertation is to define a framework for representing imperfect information, by extending crisp knowledge representation methods. By "imperfect", we refer either to uncertain or vague concepts. The general idea is to define a knowledge representation scheme, that allows for statements with uncertainty and vagueness degree conditions. This representation assigns a truth degree in the interval $[0, 1]$ rather than a true/false value. Our framework is aligned with semantic web knowledge representation frameworks and it is defined based on these theories. Thus, our approach can be defined as a "*semantic web knowledge representation approach for representing uncertain and vague concepts*".

2.2 Thesis steps - Achievements

More precisely, throughout our dissertation we have proceeded through the following steps. For each step the reached achievements are also presented:

- Propose a definition of an "imperfect" Description Logic along with an "imperfect" Ontology, that captures both uncertain and vague concepts. Towards this concept the following sub-goals have been achieved:
  - Define ontologies that capture uncertain and vague concepts: An uncertainty ontology and an entailment method which is based on Dempster-Shafer model are described and implemented.
  - Define an extension of a crisp DL with Belief - Plausibility Degrees: We propose a framework that employs Dempster-Shafer theory in a Description Logic Knowledge Base environment. More precisely, we have defined a Dempster-Shafer DL Knowledge Base, in order to represent uncertainty in a Description Logics framework. In addition, a combination method of independent Dempster-Shafer DL Knowledge Bases has been proposed, based on Dempster's rule of Combination.
  - Define an extension of a fuzzy DL with Belief Degrees: Vague information has been emerged as a main issue in Semantic Web community. Vagueness is traditionally represented by Fuzzy Set theory. Besides vagueness, Semantic Web queries often have to deal with information incompleteness, aka uncertainty. This kind of information can be represented through Dempster-Shafer theory, that also enables distributed information fusion. Imperfect information, i.e uncertainty and vagueness, should be represented and manipulated under a common framework. We propose such a framework by defining a fuzzy Description Logic extended with Dempster-Shafer theory. Furthermore, we regard our method as a DL extension and we implemented it by a meta-ontology that captures Dempster-Shafer Fuzzy statements.
- Testing and evaluating our framework in real-world case studies: In order to test our methodology in real-world environments, we have tested two application areas, recommender systems and matchmaking environments. We have collected a set of data, detect uncertain and vague pieces of evidence and proceeded by employing suitable applications for manipulating them.

Consequently, for defining a unified framework for representing uncertainty and vagueness, we decided to combine the following theories:

- Fuzzy Logic
- Dempster-Shafer Theory
- Description Logics

3 Background and Related Work

At first, web data were designed taking into account human readers, with HTML being the most used language. The problem is that HTML does not provide for *metadata*, i.e. data about data. Metadata capture the semantic regarding Semantic Web data. Towards this concept, XML language have been employed.

In general, information processing within the Semantic Web is done by *agents*. As it is referred in [1], a semantic web agent "*will receive some tasks and preferences from a person, seek information from web sources, communicate with other agents, compare information about user requirements and preferences, select certain choices, and give answers to the users*". It seems that the role of an agent actually demands a decision making mechanism, which in turn presupposes a method for handling uncertainty and vagueness tasks. They are generally characterized as pieces of software that operate autonomously and proactively. In Semantic Web, an agent usually employs the following technologies:

- Metadata
- Ontologies
- Logic

3.1 Semantic Web Layers

Generally, the Semantic Web is regarded as a set of layers that form a stack, with each layer being built on top of another. At the bottom of the stack resides *XML*, which is a language that allows for structured web data with a user-defined vocabulary. Next, there is *RDF* and *RDF Schema*. *RDF* is a data model that is employed for writing simple statements about Web objects (resources). In addition, *RDF Schema* provides for organizing Web objects into hierarchies. Though tools for writing ontologies are provided, there is a need for more advanced ontology languages. Thus, the next level is the *ontology languages*, that allow for representations of more complex relationships, through a variety of dialects. Then, there is the *Logic* layer that provides with the means for writing declarative knowledge. Next, the *Proof* layer comes that is the deductive process, along with the representation of proofs and proof validation. Finally, the *Trust* layer considers digital signatures and in general knowledge based on recommendations by trusted agents.

3.2 Ontologies

As previously mentioned, there is a need for web information to be represented in a way that is understandable by machines. To achieve this, the Semantic Web incorporates a lot of technologies, which are described in what we call a *semantic web stack*. In addition, in [9], the semantic web architecture is regarded as "two-towers" rather than a stack . *Ontologies* and *rules* are the most significant among these technologies. Generally, an ontology "*is an explicit and formal specification of a conceptualization*" [1]. That means that it is a *conceptualization of a domain* and provides a shared understanding of the domain. This term originates from philosophy and is the literal translation of the Greek word Οντολογία. As it is referred in [10] definitions for objects as well as types of objects should be provided. We can consider that an ontology consists of:

1. Types of entities that describe a specific domain
2. Properties of those entities

3.3 Description Logics

Description Logics is a family of *knowledge representation languages* and provide a way to "represent knowledge in a structured and formally well-understood way" [2]. They belong to a more general category called *description languages*. These languages allow the description of worlds providing constructors for building them [25, 2]. Generally, DLs support expressions that are built from atomic *concepts* and atomic *roles*. Each DL offers a specific level of expressiveness. DLs are a fragment First Order Logic (FOL), achieving lower complexity in expense of limited expressivity.

3.4 Uncertainty and Vagueness

Imperfect information includes uncertainty and vagueness concepts, which are described as follows:

- *Uncertainty*: It refers to situations when information incompleteness exist in order to decide about the truthness of a fact.
- *Vagueness*: It describes imprecise concepts, or concepts lacking clarity of definition

A good example of uncertainty and vagueness is given in [22], where the word "*degree*" is used to describe both uncertainty and vagueness measurements, but with different meaning. For example,

1. "*To some degree birds fly*" (uncertainty)
2. "*To some degree Jim is blond and young*" (vagueness)
3. "*Tomorrow, it will be a nice day*" (uncertainty and vagueness)

3.5 Fuzzy Logic and Fuzzy Sets

Fuzzy logic [34] is the logic of imprecision and approximate reasoning [36]. It is the framework for describing *vagueness*, by assigning truth values to linguistic variables [35] and aims at representing the human way of thinking. The general idea is that Fuzzy Sets' elements can belong to some degree to the set. More precisely, vagueness actually considers statements that are true to a certain degree, taken in the truth space [0,1]. In other words, statements are *graded*. Vagueness is associated with a set of vague concepts, e.g *low cost*. What is more is that vagueness is the result of ambiguity that describes information. For example a $100 ticket can be considered expensive for some people and low cost for others. The intuition behind the degree of membership is that the higher it is the more related is the object to the vague concept.

3.6 Dempster-Shafer Model and Dempster's rule of Combination

In the Semantic Web environment, usually, uncertainty comes as a result of ignorance, which in turn, is due to *incomplete information*. In other words, we talk about epistemic uncertainty. In those cases, the classical notion of probability cannot be considered suitable for the following reasons [7]:

1. Probability is not as good at representing ignorance.
2. An agent cannot always define probabilities for all sets of possible worlds.
3. In some cases, the computational effort demanded for probability definition, might be prohibitive.

Dempster-Shafer theory [28, 20] is considered a mathematical theory of evidence, that quantifies uncertainty in cases of ignorance and comes as a generalization of the Bayesian theory of subjective probability judgement. This theory is also known as *Theory of Belief Functions* or *Evidence Theory*. Bayesian theory quantifies judgements by assigning probabilities to the set of possible answers. Dempster-Shafer theory allows for deriving degrees of belief for a specific question based on probabilities for another related question.

4 Our Approach

As we have stated in the introduction, the Semantic Web vision introduces the concept of machine-processable information. In cases of imperfect information, i.e uncertainty and vagueness, the classical concept of ontology should be extended for capturing imperfect knowledge. Towards this concept, we aim at representing imperfect knowledge in an ontological environment.

In [18], an ontology for manipulating uncertainty, based on Dempster-Shafer theory, is described. The basic concepts of Dempster-Shafer model are represented through a Semantic Web ontology. Following, a set of entailment methods is combined through a method based on Dempster's rule of Combination.

In [19], an approach for representing uncertainty and vagueness is outlined. This approach considers vague knowledge represented through a fuzzy DL. In addition, an ontology is employed for representing information in a rule/event form, in order to perform reasoning. Both uncertainty and vagueness are represented by an *imperfection factor*. Big data processing have been also taken into account in this work.

In [15], an approach suitable for imperfect knowledge in a matchmaking case study is outlined. Matchmaking problems [13] can be considered as a case study of Semantic Web applications. In general, a matchmaking application considers a set of criteria, set by two parts. Towards this, we propose a matchmaking method of web data based on fuzzy criteria. Our method employs Dempster-Shafer theory and Dempster's rule of Combination in order to derive a combined constraint degree that represents the degree of matchmaking between the two parts (the seeker and the offer).

Following, we proposed a framework that employs Dempster-Shafer theory in a Description Logic Knowledge Base environment [16]. We name our model a *Dempster-Shafer DL Knowledge Base*.

As we have stated in the introduction, while developing Semantic Web applications, we often come across information incompleteness issues. As an example, let us consider a data source that contains information about *hotels*. We assume each hotel h to be assigned an interval cost per night rather than a crisp value, e.g:

$$h : [50 - 150]$$

In this case, if we want to make a reservation, we do not know exactly what the cost is but we know a lower-upper bound of the cost value. Moreover, consider the following query:

I'm looking for a hotel with cost no greater than 100

In a crisp logic framework, where each hotel has a unique value cost, the query could be answered with a yes/no statement. In our case, where we have to deal with interval value form, a yes/no statement cannot fully answer this query. The introduction of a *degree* notion seems to be more suitable to describe this kind of information.

In a Description Logics environment, if we consider a concept *DesiredHotel*, defined as:

$$DesiredHotel \equiv \; Hotel \sqcap \exists cost. \leq_{100}$$

then, the answer to our query is to decide whether a hotel individual is a member of the Class *DesiredHotel*.

Information incompleteness can be classified as an uncertainty problem. Dempster-Shafer theory, along with Dempster's rule of Combination [27], is a framework for dealing with information incompleteness, allowing integration of information from different independent sources. In our dissertation, we proposed an adaptation of Dempster-Shafer theory in a logic context.

More precisely, we define an extension of crisp Knowledge Bases with Dempster-Shafer modules. Dempster-Shafer Theory is more well-suited in modelling beliefs regarding the truthness of an event. Our method is an extension of the crisp DL $\mathcal{ALC}$. In our framework, we consider crisp DL axioms annotated with Dempster-Shafer belief and plausibility degree conditions.

As it is referred in the introduction, there is a need for representing uncertainty and vagueness through a common framework, especially in web-application areas. As a final step, we extended the theory of a fuzzy DL with a Dempster-Shafer framework. This framework is presented in [17, 14]. Our framework, denoted as *Dempster-Shafer Fuzzy Description Logic*, constitutes a generalization scheme of a crisp DL with fuzzy conditions along with a Dempster-Shafer module.

Taking into account the fuzzy DL interpretations introduced in [29], our framework considers any such interpretation as a *possible world*. The set of possible worlds is regarded as a frame of discernment. Thus, a basic probability assignment function is assigned on subsets of this set. This measure constitutes the uncertainty framework of our method.

A classical DL, assumes a universe $\mathcal{X}$ and subsets $\mathcal{A} \subseteq \mathcal{X}$, that constitute a DL *Concept*. Any element $x \in \mathcal{X}$ belongs to $\mathcal{A}$ or not, which is interpreted as a true/false value. The fuzzy extension assumes truthness interval on $[0, 1]$, where $\mathcal{A}$ is a *Fuzzy subset* and it is associated with a membership function $\mu_{\mathcal{A}}(x) : \mathcal{X} \rightarrow [0, 1]$. Any DL axiom, either crisp or fuzzy, has a truth value in a fuzzy interpretation $\mathcal{I}$. Our innovation, *Dempster-Shafer Fuzzy Description Logic* assigns probability masses into sets of fuzzy interpretations.

Let $\mathcal{W}$ a set of fuzzy DL interpretations. Let's denote a basic probability assignment function, m_{DS} on $2^{\mathcal{W}}$ as $m_{DS} \ : \ 2^{\mathcal{W}} \rightarrow [0, 1]$. Then, the extension of our method employs sets of fuzzy DL interpretations $\mathcal{I} \in \mathcal{W}$ in order to define Belief Degrees of fuzzy subsets of an interpretation domain $\Delta^{\mathcal{I}}$ (or $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$). This means that we assume a Fuzzy Description Logic and define Belief Degrees Conditions for axioms of this logic. In our case, we have considered the DL $\mathcal{ALC}$ and based on a fuzzy extension of it, we define our Dempster-Shafer Fuzzy DL. Since we extend fuzzy $\mathcal{ALC}$ based on Zadeh fuzzy logic, we also employ this logic in our framework.

For both our frameworks, i.e. the *Dempster-Shafer DL Knowledge Base* and the *Dempster-Shafer Fuzzy Description Logic*, we have examined decidability and complexity issues. In our approach, we adapt and extend the decidability procedure described in [29, 30] defined over fuzzy $\mathcal{ALC}$, in order to account for Dempster-Shafer Degree Conditions. This approach was first introduced in [4] in a propositional logic framework.

5 Conclusions and Future work

In our thesis, we defined an approach for representing uncertainty and vagueness under a common framework in a Semantic Web environment. In order to represent uncertainty we employed Dempster-Shafer model. Vagueness has been represented through Fuzzy Logic and Fuzzy Sets. At first, we examined our problem though an ontological point of view. Thus, we implemented suitable semantic web ontologies for capturing imperfect concepts. Following, for establishing our theoretical framework, we combined the classical crisp DL $\mathcal{ALC}$ with a Dempster-Shafer module. Next, we have proceeded by adding fuzziness in this model. Throughout our work, we formally defined the syntax and the semantics and examined decidability and complexity issues.

The main advantage of our method resides on the fact that we do not tackle uncertainty and vagueness as independent notions. This representation is in accordance with real-world applications, since very often uncertainty and vagueness coexist. The Dempster-Shafer model has been proven to be an ideal framework for representing estimations, since it models a world in a way similar to human thinking, in cases of reasoning.

In addition, our theoretical framework has been built upon $\mathcal{ALC}$, a well-established DL. Our syntax has been defined as an extension of $\mathcal{ALC}$ syntax. Vagueness is represented through Zadeh's Fuzzy Logic, by considering membership degree conditions on crisp $\mathcal{ALC}$ axioms. In addition, we employ Dempster-Shafer theory for representing the uncertainty part. In order to employ this theory, we have defined belief degree conditions. The notion of *possible world* has an important role in defining the semantics of our framework. More precisely, we have regarded the set of possible worlds, i.e. fuzzy DL interpretations, as a frame of discernment and defined mass functions on subsets of this set. As a final step, we have considered the combination of statements from different Knowledge Bases, by employing our Combined Dempster-Shafer entailment, an entailment method based on Dempster's Rule of Combination.

The Dempster-Shafer framework was proven to be an ideal one for representing ignorance. Although it has many advantages, the complexity of the rule of Combination along with conflicts' modelling remains an issue to be tackled for representing real world case studies. As a future work, we will consider complexity and decidability issues more thoroughly, mostly aiming at Dempster's rule evaluation performance. In [27], other formulas for combining evidence are outlined. These formulas provide for lower complexity. Thus, the adaptation of these formulas in a DL environment can serve as a way to gain better complexity.

We shall also consider Big Data environments in a more thorough framework. Although we examine some Big Data issues through our dissertation, we do not consider some well known algorithms such as the one defined in [5]. As a future work, we will focus on the application of our model in a Big Data environment.

Another area of future work resides in the expressiveness level. As our dissertation has been defined upon DL $\mathcal{ALC}$, we may consider the extension of other DLs. Apart from fuzzy $\mathcal{ALC}$, other fuzzy extensions are described in [29], [22], [24], [32], [30],[31]. Moreover, although $\mathcal{ALC}$ is the basic DL, in cases of Semantic Web, a set of other DLs is usually employed, namely, $\mathcal{SROIQ}(D)$ [8], $\mathcal{SHOIN}$ [11] and $\mathcal{SHIF}$ [12]. So it will be useful to extend our framework to these DLs. In addition, for representing vagueness, we employed Zadeh's Fuzzy Logic. In future, we will consider other Fuzzy Logics as well.

In cases of strongly conflicting evidence, Dempster's Rule produces counter-intuitive examples. Towards this, other rules have been proposed [27]:

- The Discount and Combine method
- Yager's modified Dempster's Rule
- Inagaki's modified Dempster's Rule
- Zhang's Center Combination Rule

As a future work, we may consider the combination of evidence based on some of these rules.

In the area of applicability, case studies other than recommender systems and matchmaking environments can be examined. Some of them are:

- Semantic annotation
- Information extraction
- Ontology alignment
- Representation of background knowledge

These fields are described in [22] as some of the most representative ones of Semantic Web applications.

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