Performance Evaluation Techniques for Modern Wireless Telecommunication Systems*

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Abstract. Wireless communication system (WCS) design confronts the challenges of fading and interference, while emerging wireless-enabled applications are even more demanding in terms of spectrum occupancy and quality of service (QoS). Addressing these challenges and emerging trends was the motivation of choosing, in this thesis, two different WCSs, one generic and one system-specific, as case studies. However, the performance evaluation of the presented modern WCSs by using conventional techniques is far too complex or time-consuming. This thesis aimed to address the problems related to the performance evaluation of such systems, by proposing novel alternative approaches than those existing in literature.

Dissertation Summary

Emerging wireless applications have different QoS requirements depending on the nature of the services offered and the expectations of the users in each case. Real-time applications impose very strict delay constraints in order to provide adequate user satisfaction. For example, remote video-conferences require almost zero delay tolerance to ensure effective communication among the participants. On the other hand, data exchange applications require very high throughput in order to disseminate, for instance, massive volumes of information, whereas they can allow certain levels of delay. Different applications have varying throughput vs delay constraints, requiring adaptability in order to address these, sometimes controversial, requirements. The varying nature of the wireless fading channels imposes additional challenges in guaranteeing a certain QoS level.

As transmit power and spectrum occupancy are limiting resources in wireless communications, several works have considered power and rate adaptation policies in order to optimize spectral efficiency. In order to quantify the performance considering these QoS aspects, the conventional notion of the Shannon or outage capacity cannot be used, as they are not considering the delay aspect. An alternative QoS metric that is able to capture the end-to-end communication delay is the effective capacity (EC), introduced in [1], is able to quantify the system performance under QoS limitations.

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The EC in wireless communications is affected by channel fluctuations due to fading and depends on the knowledge of the channel state information (CSI) at the transmitter and the receiver. If we assume CSI is available, we can consider the following power and rate adaptation policies for the transmitter that have been studied in past: i) Constant power with optimal rate adaptation (ORA); ii) Optimal power and rate adaptation (OPRA); iii) Channel inversion with fixed rate (CIFR) and iv) Truncated CIFR (TIFR).

Recently, EC performance studies have been carried out in connection with single-input single-output (SISO) systems, multiple-input single-output (MISO) systems with transmit maximal ratio combining (MRC) and relaying systems employing the amplify-and-forward (AF) protocol, e.g. see [2–15].

In [9–11], various analytical approaches for the computation of the EC of wireless systems operating over arbitrary generalized fading channels have been presented. These approaches utilize the moment generating function (MGF) to deduce single-integral expressions for the EC.

The EC of AF systems assuming ORA policy has been investigated in [12], [13], [14], [15]. However, all these approaches address the EC performance of relaying schemes considering only the ORA policy. In fact, to the best of the author's knowledge, there is no EC performance evaluation study of AF relaying operating over arbitrary fading and under different adaptive transmission schemes available in the open technical literature.

Another popular diversity scheme, namely equal gain combining (EGC), is a practical alternative to MRC, as its performance is comparable to that of MRC and at the same time it exhibits significantly lower implementation complexity [16, Chapter 9, p. 331].

Nevertheless, to the best of our knowledge, the EC of EGC receivers operating over arbitrary and/or correlated generalized fading channels has not been considered in the open technical literature yet. This is mainly because of the inherent difficulty in obtaining simple, mathematically tractable expressions for the probability density function (PDF) of the sum of fading envelopes [16, Chapter 9, p. 334]. Moreover, various performance evaluation approaches presented in the past are valid only for MRC diversity receivers and thus cannot be applied to the evaluation of the EC performance of other diversity receivers, such as the EGC.

Motivated by the above[1], in the first part of this thesis a more general approach is followed by considering a signal combining structure, whose output is the L_p-norm of the fading envelopes of each of the L diversity branches. For this structure, and by developing new MGF- and CHF-based analytical frameworks, its EC performance for different adaptive transmission policies is analyzed. This is a generic diversity structure which includes several well known diversity schemes, including EGC (L_1-norm diversity), MRC (L_2-norm diversity) and AF transmission (L_{-2}-norm diversity), as special cases. This generic receiver structure can be used to obtain the EC performance for a variety of diversity schemes operating over arbitrary fading channels by means of standard

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[1] Major part of the work described in this chapter has been published in [17, 18].

numerical integration techniques, provided that the MGF or the CHF of the p-th power of the fading envelope, where $p \in \mathbb{R}$ can be obtained. Although the presented analysis is general enough to accommodate most of the well-known fading distributions available in the open technical literature, for performance evaluation purposes, five channel fading models are considered, namely the Nakagami-m, the Gamma-shadowed generalized Nakagami-m (GSNM), the generalized gamma (GG), the α-κ-μ and the α-η-μ. Within this generic framework, the main novel contributions of this part can be summarized as follows.

- Novel, single integral generic expressions for the EC performance analysis of L_p-norm diversity receivers operating under the ORA schemes are deduced. The proposed analysis includes other previously published MGF-based approaches, where EC performance evaluation results for the MRC receiver over arbitrary generalized fading channels have been obtained, as special cases. Using these generic expressions, closed-form analytical results for the EC of dual-branch L_p-norm diversity receivers, operating under GSNM fading channels are further presented. In order to obtain further insights as to how the various system parameters, such as delay constraints, fading parameters and number of antennas, affect the EC performance, a novel asymptotic performance analysis is introduced. Using this approach, the EC of MRC and EGC diversity receivers operating under the ORA scheme at low- and high-SNR regimes can be conveniently and accurately obtained;
- Assuming operation under the OPRA scheme, novel single-integral expressions for the computation of the EC of L_p-norm diversity receivers are provided in terms of the incomplete MGF or the characteristic function (CHF) of the p-th power of the fading envelope, where p is an arbitrary real number. A computationally efficient CHF-based approach for the numerical evaluation of the optimal cutoff threshold is further presented;
- Novel analytical expressions for the EC of L_p-norm diversity receivers operating under the CIFR and TIFR schemes are deduced. These expressions are given in terms of the MGF and the CHF of the SNR at the receiver output, respectively.

Moreover, as 5G systems are adopting multi-tier architectures consisting of macrocells, different types of licensed small cells, and device-to-device networks, operation in such systems is expected to create interference issues when different tiers share the same spectrum. Within the cognitive radio concept, we can model these systems by considering two tiers. The first one, presumably the licensed, as the primary system, while the second as the secondary or unlicensed or cognitive system. The secondary system needs to be enabled with cognitive capabilities in order to reduce or eliminate interference towards the primary and thus be allowed to share the same spectrum.

Vandermonde-subspace frequency division multiplexing (VFDM), introduced in [19], is an overlay cognitive radio technique based on dynamic spectrum access (DSA). VFDM allows a single-user secondary system to operate simultaneously with the single-user primary over the same frequency band. Such operation is achieved with an appropriate precoder [20] at the secondary system that eliminates interference towards the primary, without the need for cooperation between the different tiers nor multiple transmitter/receiver antennas. This technique can be applied to multi-user two-tier block transmission systems [21], with a guard interval or a cyclic prefix, over frequency-selective (FS) channels. The performance of a single-user (per tier) scenario has been evaluated by means of Monte Carlo computer simulations in [22], where an expression for the SINR at the secondary receiver was also presented. However, this expression cannot be used to obtain analytical results, and the computer simulations carried out in [22] are typically very time-consuming.

In the second part of this thesis[2], an alternative approach is proposed, namely the analytical approximation of the SINR statistics of the considered cognitive system by the gamma distribution. The advantages of this approach are threefold: *i)* Although other distributions have been considered, it turns out that the gamma distribution accurately approximates the SINR statistics over a wide-range of system implementations; *ii)* it is the only accurate distribution that allows the derivation of novel closed-form expressions for the ergodic capacity and average bit error probability (ABEP); and *iii)* it facilitates the performance evaluation of different practical implementations, including those that use industrial standards, by deriving the gamma distribution parameters as functions of the system operating parameters.

Indicative Results and Discussion

Performance analysis of generalized diversity receivers

We consider an L-branch diversity receiver operating in the presence of arbitrary generalized fading and additive white gaussian noise (AWGN). A generic analytical expression for the instantaneous SNR, γ_{end}, at the output of the L_p-norm diversity receiver can be deduced as [25, Eq. (2)], [26],

$$\gamma_{\mathrm{end}}(\vec{\mathbf{R}}) = \mathrm{K}\frac{E_s}{N_0}\left(\sum_{\ell=1}^{L}\mathcal{R}_\ell^p\right)^q, \tag{1}$$

where E_s/N_0 is the SNR per symbol, with E_s being the average symbol energy and N_0 the single-sided power spectral density of the additive white Gaussian noise (AWGN), q is a real number and K is a constant that depends on the diversity combining scheme. For example, in (1), for $(p, q, \mathrm{K}) = (1, 2, 1/L)$, $(p, q, \mathrm{K}) = (2, 1, 1)$ and $(p, q, \mathrm{K}) = (-2, -1, 1)$, the EGC, MRC and AF transmission schemes are obtained, respectively.

Assuming block fading channels, the EC is defined as [1]

$$R(\theta) \triangleq -\lim_{N\to\infty}\frac{1}{N\,\theta\,T\,B}\ln\left[\mathbb{E}\left\langle\exp\left(-\theta\sum_{n=1}^{N}S[n]\right)\right\rangle\right], \tag{2}$$

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[2] Major part of the work described in this chapter has been published in [23] and [24]

where $\{S[n], n = 1, 2, \ldots\}$ is a stationary and ergodic stochastic process that characterizes the service rate at the source node, θ is the asymptotic decay-rate of the buffer occupancy, B is the system bandwidth and T the fading block length.

Considering all four adaptive transmission policies and assuming arbitrarily distributed $\mathcal{R}_\ell$, the analytical evaluation of the EC involves the numerical evaluation of L-fold integrals, whose exact analytical expressions depend upon the employed adaptive transmission policy.

An MGF-based approach for the evaluation of the EC of diversity receivers under the ORA policy can be obtained as follows. Under the ORA policy, the EC of L_p-norm diversity receivers over arbitrary not necessarily independent nor identically distributed generalized fading channels can be expressed in terms of a single integral as

$$R^{\mathrm{ORA}}(\theta) = \begin{cases} -\frac{1}{\theta T B} \ln \left[\int_0^\infty C_q(u) \mathcal{M}_{\vec{\mathbf{R}}^p}(\mathrm{k}u) \mathrm{d}u\right], \; q > 0 \\ \\ -\frac{1}{\theta T B} \ln \left[- \int_0^\infty C_q(u) \frac{\partial \mathcal{M}_{\vec{\mathbf{R}}^p}(\mathrm{k}u)}{\partial u} \mathrm{d}u\right], \; q < 0 \end{cases} \tag{3}$$

where $\mathcal{M}_{\vec{\mathbf{R}}^p}(u) = \mathbb{E}\langle \exp(-u \sum_{\ell=1}^{L} \mathcal{R}^p)\rangle$ is the joint MGF of the p-th exponent of the random vector $\vec{\mathbf{R}}$, $k = \mathrm{K} E_s/N_0$ and the function $C_q(u)$ is given by

$$C_q(u) = \begin{cases} \frac{u^{-1}}{q\Gamma(A)} H_{1,2}^{1,1}\left[u \left| \begin{smallmatrix} (1,1/q) \\ (A,1/q),(1,1) \end{smallmatrix} \right.\right], & q > 0 \\ \frac{1}{|q|\Gamma(A)} H_{1,2}^{1,1}\left[u \left| \begin{smallmatrix} (1-A,1/|q|) \\ (0,1/|q|),(0,1) \end{smallmatrix} \right.\right], & q < 0. \end{cases} \tag{4}$$

The accuracy of the proposed analytical framework is substantiated with numerical results, accompanied with equivalent performance evaluation results obtained by means of Monte-Carlo simulations.

The α-κ-μ is a very general fading model that includes as special cases several well-known distributions, namely the GG, the κ-μ, the Nakagami-m and the Rice distribution. For the α-κ-μ distribution the PDF of $\mathcal{R}_\ell$ is given by Eq.**??**

An exact closed-form expression for the MGF of $\mathcal{R}_\ell^p$ can be obtained in terms of the bivariate Fox's H-function, which can be evaluated numerically using the Matlab code presented in [27]. An alternative computationally efficient expression for $\mathcal{M}_{\mathcal{R}_\ell^p}(u)$ can be obtained as

$$\begin{aligned} \mathcal{M}_{\mathcal{R}_\ell^p}(u) &= \frac{2\alpha\mu}{p} \exp[-\mu\kappa] \kappa^{(1-\mu)/2} (1+\kappa)^{(1+\mu)/2} u^{-\frac{\alpha(1+\mu)}{2p}} \\ &\times \sum_{k=1}^{N_t} w_k {t_k}^{\alpha(1+\mu)/p-1} I_{\mu-1}\left[2\sqrt{\kappa(1+\kappa)}\mu \left(\frac{t_k^2}{u}\right)^{\alpha/p}\right] \exp\left[-\mu(1+\kappa)\left(\frac{t_k^2}{u}\right)^{\alpha/p}\right], \end{aligned} \tag{5}$$

where the number of integration points N_t, the weights w_k and abscissae t_k are given in [28].

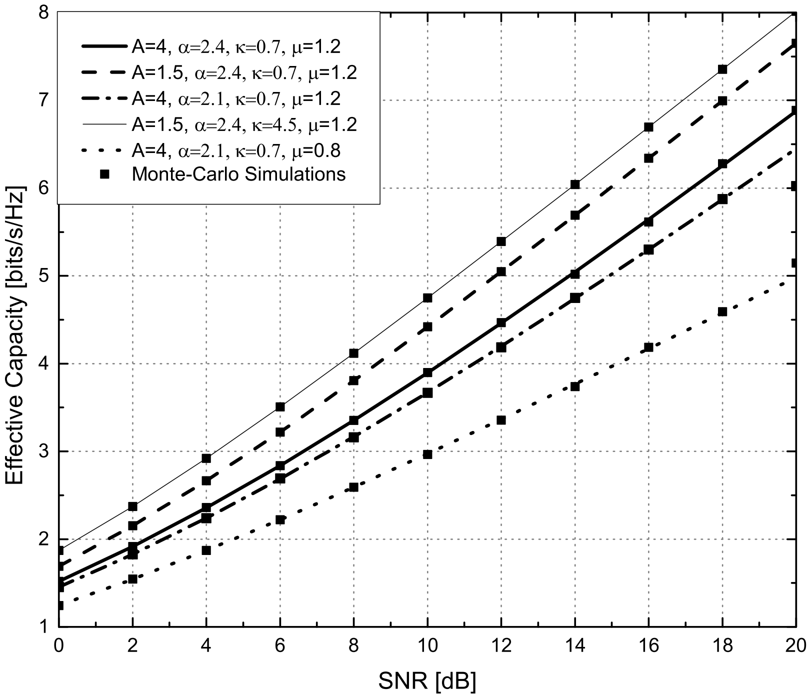
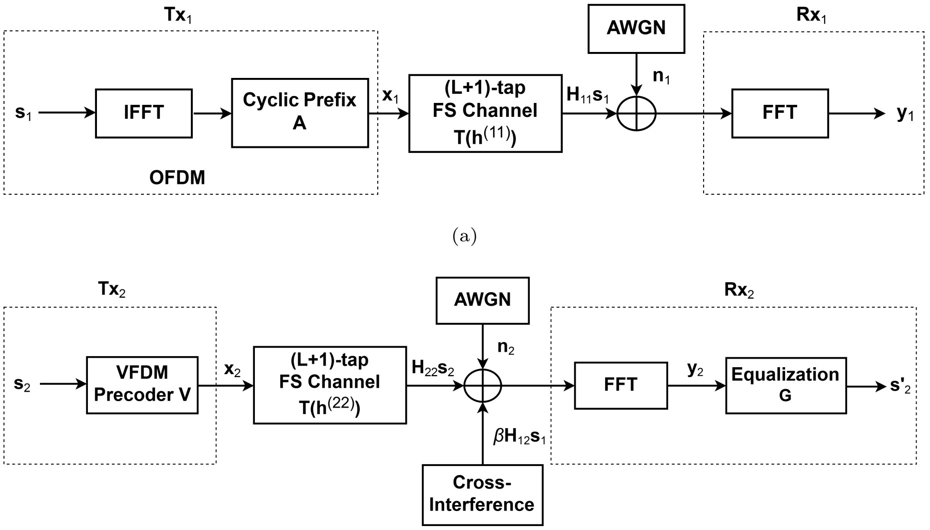


Fig. 1. Exact analytical EC performance evaluation results for EGC receivers with $L = 3$ branches operating over i.i.d. α-κ-μ fading under the ORA policy

Fig. 1 depicts the EC performance of an EGC receiver with $L = 3$ diversity branches, under i.i.d. α-κ-μ fading and, as it can be observed, analytical and simulated performance evaluation results are in excellent agreement.

Performance analysis of cognitive VFDM systems

The detailed system model under consideration is depicted in Fig. 2, where the primary and secondary systems are presented in Figs. 2a and 2b, respectively.The primary system consists of a primary user with a transmitter Tx_1, which utilizes the Orthogonal Frequency Division Multiplexing (OFDM) with N subcarriers and a cyclic prefix of size L. The secondary system consists of a secondary user with a transmitter Tx_2 that uses the VFDM technique. More specifically, the secondary user exploits the resources created by the frequency selectivity of the channel and the cyclic prefix used by the primary by implementing an $(N + L) \times N$ precoder $\mathbf{V}$ that eliminates its interference towards the primary receiver Rx_1.



(a)

(b)

Fig. 2. Block diagram of the two-tier cognitive VFDM system model: (a) Primary system (b) Secondary (cognitive) system.

As the primary system's operation is not affected by the presence of the secondary, it can be considered as a typical OFDM system, which has been extensively studied in the past. The SINR at the secondary receiver can be rewritten in a more general form, as compared to [22, Eq. (11)], as

$$\gamma_k = \frac{|\mathbf{g}_k \mathbf{h}_{2k}|^2}{\sigma^2 |\mathbf{g}_k \mathbf{g}_k^H|^2 + \beta^2 \sum\limits_{m=1}^{N} |\mathbf{g}_k \mathbf{h}_{1m}|^2 + \sum\limits_{n=1; n \neq k}^{L} |\mathbf{g}_k \mathbf{h}_{2n}|^2}, \tag{6}$$

where γ_k is the SINR of the k^{th} received symbol, $\mathbf{h}_{ik}$ is the k^{th} column of $\mathbf{H}_{i2}$ and $\mathbf{g}_k$ is the k^{th} row of the equalization filter $\mathbf{G}$ that is applied at the receiver so that we get the estimated symbols

$$\mathbf{s}_2' = \mathbf{G}\mathbf{y}_2.$$

An important performance metric of communication systems is the (ergodic) capacity, which can be calculated as

$$C_{erg} = \frac{1}{L} \sum_{k=1}^{L} \mathbb{E} \left\langle \log_2 (1 + \gamma_k) \right\rangle . \tag{7}$$

However, the above computation involves multiple sums of large matrices, which might not produce closed-form expressions, while obtaining results via computer simulations is typically very time-consuming. Alternatively, one can estimate the capacity as

$$\widetilde{C}_{erg} = \int_{0}^{\infty} \log_2(1+x) f(x;a,b)dx, \tag{8}$$

where $f(x;a,b)$ is the approximate PDF of the SINR in (6).

In this case, by using the gamma distribution [29, 30], whose PDF is given by

$$f(x;a,b) = \frac{1}{b^a \Gamma(a)} x^{a-1} \exp(-x/b), \tag{9}$$

where a is the shape parameter and b is the rate parameter, as the analytical approximation of the PDF of (6) and with the help of [31, Eq. (3)], the capacity can be obtained in closed-form as

$$\widetilde{C}_{erg}^{CF} = \frac{1}{\ln(2)\Gamma(a)b^a} G_{2,3}^{3,1}\left[\frac{1}{b}\middle| \begin{matrix} -a, 1-a \\ 0, -a, -a \end{matrix}\right]. \tag{10}$$

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In order to obtain analytical expressions for the parameters (a,b) as a function of the system parameters (N,L), linear and non-linear regression techniques were tried. As linear functions failed to fit the data, the non-linear regression (NLR) technique was applied. With this technique, successive approximations were made in order to model the observational data with a heuristic function that is a nonlinear combination of some independent variables. Thus, by applying NLR in Matlab on the previously created data set of the MLE parameters, the following approximate expressions were empirically derived for the pair (a,b), denoted in this case as $(\widetilde{a},\widetilde{b})$, in terms of (N,L)

$$\widetilde{a} \approx \frac{c_1}{N} + \frac{c_2}{L} + c_3 \text{ and } \widetilde{b} \approx \frac{c_4}{N} + \frac{c_5}{L} + c_6, \tag{11}$$

where $\widetilde{a}$ and $\widetilde{b}$ denote the parameters estimated with the NLR technique and c_1-c_6 are coefficients whose values depend on the cross-interference and SNR levels. Note that these expressions are valid for both ZF and MMSE equalization filters. The values for c_1-c_6 with cross-interference factors $\beta = \{0, 0.1, 0.3, 1\}$ and SNR values from -4 up to 20 dB (in steps of 2 dB) have been obtained and can be found in tabulated form in [24].

In order to validate the proposed analysis, the performance of a secondary user under the considered VFDM cognitive spectrum sharing system has been evaluated and the analytical results were complemented with equivalent performance evaluation results obtained by means of Monte Carlo computer simulations.

A VFDM secondary user operating simultaneously with a primary user in an IEEE 802.11 a/g (WLAN) system [32] $(N = 64, L = 16)$ was considered. For this case, the capacity performance of the secondary user for cross-interference factor values $\beta = \{0, 0.1, 0.3\}$ using the ZF equalization filter is depicted in Figs. 3. The results show that for all cases considered, there is a fine agreement between the approximated and simulated performance results, confirming the accuracy of the proposed approximation. It is also noted that the values of the closed-form expression always coincide with the values of the integral expression for the capacity.

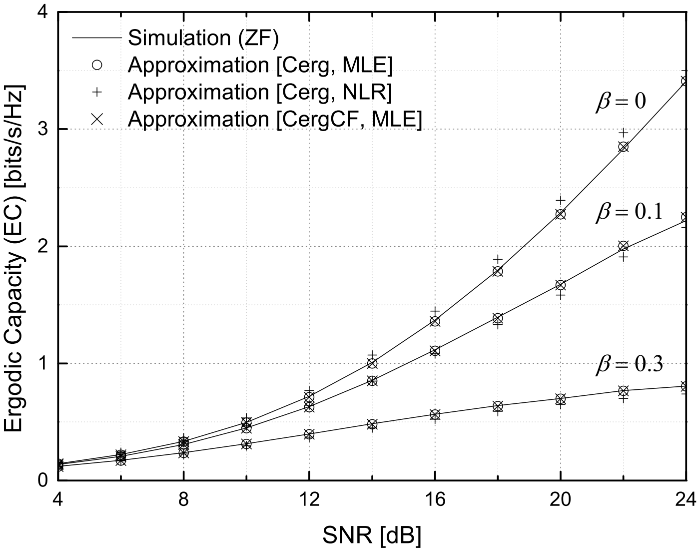

Fig. 3. Capacity vs. SNR performance for the IEEE 802.11 a/g standard with ZF equalization, for the various methods of evaluation under consideration and different values of β.

Conclusions

In this section, the main contribution of this thesis is summarized. In the first part, a real-time generic diversity combining system that operates over generalized fading channels under different adaptive transmission schemes was studied. In order to take into account the delay-sensitivity of such real-time applications, an alternative performance metric rather than the conventional Shannon or outage capacity is required. Lately, the EC has attracted attention as a suitable metric quantifying end-to-end system performance under QoS limitations.

Therefore, in this thesis, a novel analytical framework for obtaining the exact effective capacity (EC) performance of L_p-norm diversity reception over arbitrary generalized fading channels and different adaptive transmission policies, namely Optimal Power and Rate Adaptation (OPRA), Optimal Rate Adaptation (ORA), Channel Inversion with Fixed Rate (CIFR) and truncated CIFR (TIFR), was proposed. For the special case of dual diversity, closed-form expressions for the EC performance over gamma-shadowed generalized Nakagami-m (GSNM) fading channels have further been deduced. Finally, an analytical moment generating function (MGF)-based approach for the asymptotic analysis of the EC performance at low- and high-SNR regions was also proposed thus providing useful insights regarding the operating parameters which affect the overall system performance. The validity of the proposed analytical methodology was assessed by considering very generic channel fading models that describe wireless propagation in a more realistic manner than the conventional fading models. The accuracy of the proposed analysis was substantiated with numerical results, accompanied with equivalent performance evaluation results obtained by means of Monte-Carlo simulations.

In the second part of this thesis, a two-tier Vandermonde-subspace Frequency Division Multiplexing (VFDM) cognitive system operating over Rayleigh fading that allows unlicensed users utilize the licensed spectrum was considered. In this thesis the approximation of the signal to interference plus noise ratio (SINR) statistics of such a system by the gamma distribution was proposed. Although other distributions were considered, the gamma distribution provided the best fitting results. Moreover, it was shown that using this approximation, closed-form expressions for the capacity and the average bit-error probability (ABEP) can be derived, facilitating the performance evaluation. Furthermore, analytical expressions were provided for obtaining the parameters of the gamma distribution as functions of the system operating characteristics. The accuracy of the proposed analysis was validated with analytical and complementary computer simulation performance evaluation results for such VFDM cognitive spectrum sharing systems employing the IEEE 802.11 a/g and 3GPP LTE standards.

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