Nonlinear signal processing applied to telecommunications

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Abstract. Nonlinear behaviour appears in almost all digital communication systems, such as satellite systems, telephone channels, mobile cellular communications, wireless LAN devices, radio and TV channels, digital magnetic systems, etc. Linear approximations that do not take into account this type of behaviour, may lead to system performance degradation as well as loss of information. Therefore, appropriate models should be developed that tackle nonlinear system characteristics. Another important issue in studying both linear and nonlinear systems is that of the order (memory length) of the associated subsystems. It is critical, because knowing the exact subsystem orders may lead to accurate system identification and channel equalization. The primary objective of this dissertation is the solution of the structure determination problem in system identification. More specifically, the following two main objectives are pursued:

- To develop methods and algorithms for the order determination of both linear and nonlinear systems.
- To develop methods and algorithms for system identification and channel equalization.

1 Introduction

Equalization or deconvolution is essentially a signal processing procedure to restore a set of source signals which were distorted by an unknown linear or nonlinear system, whereas system identification is a signal processing procedure to identify and estimate the unknown linear or nonlinear system. The two problems prove important in a variety of areas of telecommunication applications. In this dissertation, we study blind equalization and identification methods, not only for linear systems but also for nonlinear systems modelled by finite Volterra series [1]. The approach we take towards blind identification/equalization is the following. We develop a method that identifies the orders (memory lengths) of the discrete subsystems that comprise the total system (either linear or nonlinear). This method is implemented by a computationally efficient algorithm. The algorithm detects the different subsystem orders as well as the number of

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subsystems that attain the same order. This is done both for Single Input Multiple Output (SIMO) and Multiple Input Multiple Output (MIMO) Linear Time Invariant (LTI) Finite Impulse Response (FIR) systems as well as for nonlinear SIMO Volterra systems [2, 3] .

Once the orders have been determined, subsystems are clustered in groups of the same memory length. For linear systems, Blind Source Separation techniques (BSS) are used to identify the system kernels. For nonlinear Volterra systems equalization is performed [3]. The complexity of the algorithm that is introduced depends on the number of subsystems that comprise the total system, the number of output channels, the degree of nonlinearity, the number of output samples and finally the statistical properties of the input. Our effort is concentrated on developing an algorithm that minimizes complexity with respect to all the above parameters.

This summary is organized as follows; First, in section 2 the model with the underlying assumptions, the data structures used and the order determination algorithm are presented. Section 3 provides the kernel identification algorithm for LTI FIR MIMO systems and the equalization algorithm for SIMO Volterra systems. In Section 4, conclusions and future work are presented.

2 Order Determination

Given an LTI FIR system or a nonlinear system described by a finite Volterra series, our objective is to establish that

(a) The number of discrete inputs can be computed.

(b) The orders (memory lengths) of the subsystems that comprise the total system are determined.

(c) Computations can be carried out by an efficient algorithm. Specifically, if $L_1, L_2, \cdots, L_P$ are the different subsystem orders, they may be grouped into r distinct numbers J_1, J_2, $\cdots$, J_r such that $J_1 < J_2 < \cdots < J_r$. Then for all i, $1 \leq i \leq r$, the number m_i of subsystems that have order J_i can be computed by the proposed algorithm.

2.1 LTI FIR MIMO Systems

We shall be concerned with FIR MIMO systems of the form:

$$\mathbf{x}(k) = \sum_{i=1}^{P} \sum_{j=0}^{L_i} \mathbf{h}_i(j) s_i(k-j) \tag{1}$$

The system has P inputs and M outputs. Thus the output signal $\mathbf{x}(k)$ is an $M \times 1$ dimensional vector. The input sequences consist of the signals $s_1(k)$, $s_2(k)$, $\cdots$, $s_P(k)$. The orders of the P subsystems are given by the integers $L_1, L_2, \cdots, L_P$. For each $1 \leq i \leq P$, and $0 \leq j \leq L_i$, $\mathbf{h}_i(j)$ is the corresponding $M \times 1$ kernel tap.

Memory lengths of the P subsystems are given by the integers $L_1, L_2, \cdots, L_P$. In the general case, some of the above integers may be equal to each other. Therefore, the r distint integer values appearing in the set $L_1, L_2, \cdots, L_P$ that denote the different subsystem orders, will in the following be denoted by J_1, J_2, $\cdots$, J_r and we shall assume, without loss of generality, that $J_1 < J_2 < \cdots < J_r$. In addition, for all i, $1 \leq i \leq r$, m_i will denote the number of subsystems attaining the same order J_i.

2.2 SIMO Volterra Systems

The nonlinear systems under consideration are assumed to be SIMO, discrete-time, time-invariant, causal and of finite memory. Furthermore, we assume that any small changes to the system's input $s(n)$ result in small changes in the system's output. Any such system can be approximated over a uniformly bounded set of input signals by a truncated Volterra series expansion of finite order P.

The output $y(n)$ of a real valued SIMO Volterra system, as stated in [1], can be described as:

$$x(n) = \mathbf{h}_0 + \sum_{p=1}^{P} \sum_{m_i=0}^{N-1} \cdots \sum_{m_p=0}^{N-1} \mathbf{h}_p(m_1, \cdots, m_p) s(n-m_1) \cdots s(n-m_p) + \eta(n) \quad (2)$$

As stated in [1], for a narrowband communication system input-output relationship is described by the equation:

$$x(n) = \mathbf{h}_0 + \sum_{p=1}^{P} \sum_{m_i=0}^{N-1} \cdots \sum_{m_p=0}^{N-1} \mathbf{h}_p(m_1, \cdots, m_{2k+1}) s(n-m_1) \cdots s(n-m_{k+1}) s^*(n-m_{k+2}) \cdots s^*(n-m_{2k+1}) + \eta(n) \quad (3)$$

2.3 Model Equivalence and Assumptions

We may cast a nonlinear system described by a finite Volterra series as a linear MIMO system. It should be noticed that there is no physical equivalence of the two types of systems since the inputs of the casted Volterra system are products of the original input. However, the casting is useful for algebraic manipulations. Due to the casting, any of the systems under consideration can be described as:

$$\mathbf{x}(k) = \sum_{i=1}^{B} \sum_{j=0}^{L_i} \mathbf{h}_i(j) s_i(k-j) \quad (4)$$

or equivalently

$$\mathbf{x}(k) = [\mathbf{H}(z)]\mathbf{s}(k) \tag{5}$$

where $[\mathbf{H}(z)]$ is the system transfer function and $\mathbf{s}(k) = [s_1(k) \cdots s_P(k)]^t$.

If L_q, $1 \leq q \leq P$, denotes the maximum of $L_1, L_2, \cdots, L_P$, then the channel polynomial matrix $\mathbf{H}(z)$ can be written as

$$\mathbf{H}(z) = \sum_{i=0}^{L_q} \mathbf{H}(i) z^{-i} \tag{6}$$

with

$$\mathbf{H}(i) = [\mathbf{h}_1(i)\mathbf{h}_2(i) \cdots \mathbf{h}_P(i)] \tag{7}$$

Again, the r distint integer values that denote the different subsystem orders, will in the following be denoted by J_1, J_2, $\cdots$, J_r and we shall assume, without loss of generality, that $J_1 < J_2 < \cdots < J_r$. In addition, for all i, $1 \leq i \leq r$, m_i will denote the number of subsystems attaining the same order J_i.

For LTI FIR MIMO systems the following assumptions are made:
A1) The input sequences $s_1(k), s_2(k), \cdots, s_P(k)$ are stationary Independent Identically Distributed (I.I.D.) zero mean signals of finite variance that are mutually independent with each other.
A2) An upper bound L of the subsystems' orders is known.
A3) The number of inputs P is strictly less than the number of outputs M. Furthermore, the channel polynomial matrix $\mathbf{H}$(z) is irreducible and column reduced.

For SIMO Volterra systems the following assumptions are made:
B1) The input sequence $s(n)$ is zero mean, i.i.d., with values in a finite alphabet of at least $P + 1$ complex numbers. Examples include PSK or QAM signals.
B2) The system transfer matrix $\mathbf{G}$(z) is irreducible. This guarantees that there are no common zeros in the (FIR) transfer functions of every pair of the subchannels involved. It is a common assumption in all methods based on Second Order Statistics.
B3) The memory of the linear kernel is strictly greater than the memory of any nonlinear term. Furthermore, the first element of the zero-th tap of this kernel is equal to unity.
B4) An upper bound L of B is known.

2.4 Definition of Data Structures

Assuming that $k, Q \in \mathcal{N}$and $Q > k$, we collect successive output vectors in the matrix

$$\mathbf{T}(k) = [\mathbf{x}(k) \quad \mathbf{x}(k+1) \quad \cdots \mathbf{x}(Q)] \tag{8}$$

Likewise, we combine successive input values in row vector form as follows

$$\begin{aligned}\mathbf{s}_1(k) &= [s_1(k) \quad s_1(k+1) \quad \cdots s_1(Q)] \\ &\vdots \quad \vdots \\ \mathbf{s}_B(k) &= [s_B(k) \quad s_P(k+1) \quad \cdots s_B(Q)]\end{aligned} \tag{9}$$

Using a window of length w, we form the following array of output samples:

$$\mathbf{X}_{k,w} = \begin{pmatrix} \mathbf{T}(k) \\ \vdots \\ \mathbf{T}(k-w+1) \end{pmatrix} \tag{10}$$

$\mathbf{X}_{k,w}$ is the data matrix defined by stacking w consecutive such observations, starting with $\mathbf{T}(k)$ and going back to $\mathbf{T}(k-w+1)$. Similarly, for all $j, 1 \leq j \leq B$, we have

$$\mathbf{S}^j_{k,w} = \begin{pmatrix} \mathbf{s}_j(k) \\ \vdots \\ \mathbf{s}_j(k-w+1) \end{pmatrix} \tag{11}$$

Next, we consider the row spaces

$$\mathcal{X}_{k,w} = \mathcal{R}\left(\mathbf{X}_{k,w}\right) \tag{12}$$

$$\mathcal{S}^j_{k,w} = \mathcal{R}\left(\mathbf{S}^j_{k,w}\right) \tag{13}$$

Finally, for any $l \in \mathcal{N}$ we define the vector space

$$\begin{aligned}\dot{\mathcal{S}}_{k,l} = &\left(\mathcal{S}^1_{k-1,L_1+w} \oplus \cdots \oplus \mathcal{S}^B_{k-1,L_B+w}\right) \\ &\bigcup \left(\mathcal{S}^1_{k+l+w,L_1+w} \oplus \cdots \oplus \mathcal{S}^B_{k+l+w,L_B+w}\right)\end{aligned} \tag{14}$$

If we think of l as a smoothing window, we see that $\dot{\mathcal{S}}_{k,l}$ is a subspace constructed from both past and future data observations. Based on the above, we state the following theorem that establishes the isomorphic relationship between input and output subspaces.

Theorem 1: For all $w \geq w_0$, $\mathcal{X}_{k,w} \doteq \mathcal{S}^1_{k,L_1+w} \oplus \cdots \oplus \mathcal{S}^B_{k,L_B+w}$.

Then, we define the projection error matrix and the kernel-input product matrices that are necessary to the development of the algorithm. The projection error matrix $\mathbf{E}_{k,l}$ is defined as

$$\mathbf{E}_{k,l} = \begin{pmatrix} \mathbf{T}(k+l) - \mathbf{T}(k+l)_{|\dot{\mathcal{S}}_{k,l}} \\ \mathbf{T}(k+l-1) - \mathbf{T}(k+l-1)_{|\dot{\mathcal{S}}_{k,l}} \\ \vdots \\ \mathbf{T}(k) - \mathbf{T}(k)_{|\dot{\mathcal{S}}_{k,l}} \end{pmatrix}$$

$\mathbf{E}_{k,l}$ is a $(M(l+1))\times(Q-k+1)$ matrix. Each block entry $\mathbf{T}(k+m)-\mathbf{T}(k+m)_{|\dot{\mathcal{S}}_{k,l}}$ of $\mathbf{E}_{k,l}$ is formed by the error resulting from the projection of $\mathbf{T}(k+m)$ on the space $\dot{\mathcal{S}}_{k,l}$ generated by past and future input values. In the following analysis,we shall fix k, without loss of generality since the carried out analysis is valid for any choice of it. For all $i, 1 \leq i \leq r$, let $\mathbf{h}_{i_1}, \mathbf{h}_{i_2}, \cdots, \mathbf{h}_{i_{m_i}}$, be the m_i subsystems that attain order J_i. Denote by $s_{i_1}, s_{i_2}, \cdots, s_{i_{m_i}}$ the inputs to the above subsystems.

Then, for a given subsystem $\mathbf{h}_{i_s}$, for all $l, J_1 \leq l \leq L$ and $t, 1 \leq t \leq m_i$, we define the matrix:

$$\mathbf{D}_{l,J_i}(\mathbf{h}_{i_t}) = \begin{pmatrix} \mathbf{h}_{i_t}(0) & \cdots & \mathbf{0}_{M\times 1} \\ \vdots & \ddots & \vdots \\ \mathbf{h}_{i_t}(J_i) & \vdots & \mathbf{h}_{i_t}(0) \\ \vdots & \ddots & \vdots \\ \mathbf{0}_{M\times 1} & \cdots & \mathbf{h}_{i_t}(J_i) \end{pmatrix},$$

Next, for all subsystems attaining the same order J_i, we form the matrix:

$$\mathbf{G}_{l,J_i} = \left(\mathbf{D}_{l,J_i}(\mathbf{h}_{i_1}) \cdots \mathbf{D}_{l,J_i}(\mathbf{h}_{i_{m_i}})\right),$$

Collect input values of the above subsystems, to form the matrix:

$$\tilde{\mathbf{S}}_{l,J_i} = \begin{pmatrix} \mathbf{s}_{i_1}(k+l-J_i) \\ \vdots \\ \mathbf{s}_{i_1}(k) \\ \vdots \\ \mathbf{s}_{i_{m_i}}(k+l-J_i) \\ \vdots \\ \mathbf{s}_{i_{m_i}}(k) \end{pmatrix}$$

Finally, for all $i, 1 \leq i \leq r$ and for all l, $J_i \leq l \leq L$ we define the matrices

$$\mathbf{\Psi}_{l,J_i} = \mathbf{G}_{l,J_i}\tilde{\mathbf{S}}_{l,J_i} \tag{15}$$

Having defined the projection error matrix and the kernel-input product matrices we proceed to:

Theorem 2

(i) For any $l : l < J_1$, $\mathbf{E}_l = 0$

(ii) For $l : J_1 \leq l < J_2$,

$$\mathbf{E}_l = \mathbf{\Psi}_{l,J_1} \tag{16}$$

(iii) For $n : 3 \leq n \leq r$ and $l : J_{n-1} \leq l < J_n$,

$$\mathbf{E}_l - \sum_{m=1}^{n-2} \mathbf{\Psi}_{l,J_m} = \mathbf{\Psi}_{l,J_{n-1}} \tag{17}$$

(iv) For $l : J_r \leq l \leq L$,

$$\mathbf{E}_l - \sum_{m=1}^{r-1} \mathbf{\Psi}_{l,J_m} = \mathbf{\Psi}_{l,J_r} \tag{18}$$

Next we establish the ranks of the projection error matrices $\mathbf{E}_l$.
Theorem 3: The following statements are true:
(i) For any $l : l < J_1$, $rank(\mathbf{E}_l) = 0$
(ii) For $l : J_1 \leq l < J_2$,

$$rank(\mathbf{E}_l) = (l - J_1 + 1)m_1 \tag{19}$$

(iii) For $n : 3 \leq n \leq r$ and $l : J_{n-1} \leq l < J_n$,

$$rank(\mathbf{E}_l) = \sum_{i=1}^{n-1}(l - J_i + 1)m_i \tag{20}$$

(iv) For $l : J_r \leq l \leq L$,

$$rank(\mathbf{E}_l) = \sum_{i=1}^{r}(l - J_i + 1)m_i \tag{21}$$

Theorem 3 indicates how to compute the system's different orders, as well as the number of subsystems that attain it in a straightforward manner. Indeed, starting with $l = 0$ we compute $rank(\mathbf{E}_0)$ and we increase l by one until $rank(\mathbf{E}_l) > 0$. Equation (19) suggests that this value of l equals the smaller of the orders J_1. Moreover, it also gives the number m_1 of the subsystems that attain it. Having determined m_1, we increase l in steps of one. As long as $l < J_2$, $rank(\mathbf{E}_l)$ remains a multiple of m_1. When this stops to hold (20) suggests that $l = J_2$. At this point having computed m_1, J_1 we use (20) to determine the number m_2 of the subsystems that attain the order J_2. We continue increasing l. As long as $l < J_3$, $rank(\mathbf{E}_l)$ increases by $m_1 + m_2$ each time l increases by one. Again, when this stops to hold, (20) suggests that $l = J_3$. At this point, having computed J_1, m_1, J_2, m_2 we use (20) to determine the number m_3 of the subsystems that have order equal to J_3. We keep increasing l by one until we reach L and proceed in the same way, using (20) and (21) to determine J_i, m_i for all $i : 1 \leq i \leq r$. When $l = L$, we compute the number P of input signals as:

$$P = \sum_{i=1}^{r} m_i \tag{22}$$

3 LTI FIR MIMO Systems Kernel Identification and Volterra Systems Equalization

Given an LTI FIR system or a nonlinear system described by a finite Volterra series the objective of the algorithm is

(a) To identify the kernels of the system in case it is LTI FIR MIMO.

(b) To equalize input symbols in case of SIMO Volterra systems.

The model as well as the assumptions used have already been defined in the previous chapter. An extra assumption is required for SIMO Volterra systems. Specifically, the memory of the linear kernel is strictly greater than the memory of any nonlinear term, while the first element of the zero-th tap of this kernel is assumed to be equal to unity. Data structures required, have already been defined in the previous section. In this section we state and prove a number of lemmas and theorems that establish the validity of the algorithm. In addition, the steps of the algorithm are presented.

Assuming k is fixed, we shall denote $\mathbf{E}_{k,l}$ by $\mathbf{E}_l$, to simplify notation. We state theorem 4 that is essential to the development of the proposed algorithm:

Theorem 4

For all $i, \quad 1 \leq i \leq r$ and each $l, \quad J_i \leq l \leq L$, the elements of each one of the matrices $\mathbf{\Psi}_{l,J_i}$ can be computed from the elements of the matrices $\mathbf{E}_{J_1}, \mathbf{E}_{J_2}, \cdots, \mathbf{E}_{J_i}$.

3.1 Algorithm Description

A batch algorithm that performs system identification/equalization is developed. For LTI FIR MIMO systems the algorithm uses the Joint Diagonilization Principle through the JADE algorithm as described in [9]. The outline of the algorithm is the following.

A. **Order Estimation**

- Step A_1: At the receiver, group the collected symbols in blocks of Q symbols per block.
- Step A_2: Using the first block, for all $l, J_1 \leq l \leq L$ compute the matrices $\mathbf{E}_l$. Store the matrices.
- Step A_3: Compuite the orders $J_1, J_2, \cdots J_r$ of the different subsystems that comprise the overall system following the algorithm established in [12]. Store $J_1, J_2, \cdots J_r$.
- Step A_4: For FIR LTI MIMO systems execute part B of the algorithm, for SIMO Volterra systems execute part C.

B. **LTI FIR MIMO Systems Identification**

- Step 1: Use matrices $\mathbf{E}_l$ computed in part $\mathbf{A}$, to compute the matrices $\mathbf{\Psi}_{l,J_i}$, according to theorem 1.
- Step 2: Set $l = J_1$. Using the equation

$$\mathbf{E}_{J_1} = \mathbf{\Psi}_{J_1,J_1} \tag{23}$$

  and the JADE algorithm identify the kernels corresponding to the subsystems of order J_1 as columns of the matrix $\mathbf{G}_{J_1,J_1}$.
- Step 3: Set $l = J_2$. Using the equation

$$\mathbf{E}_{J_2} - \mathbf{\Psi}_{J_2,J_1} = \mathbf{\Psi}_{J_2,J_2} \tag{24}$$

and the JADE algorithm identify the kernels that correspond to subsystems of order J_2 as columns of the matrix $\mathbf{G}_{J_2,J_2}$.

$\vdots$

– Step $r+1$: Set $l = J_r$. Use equation

$$\mathbf{E}_{J_r} - \sum_{i=1}^{r-1} \mathbf{\Psi}_{J_r,J_i} = \mathbf{\Psi}_{J_r,J_r} \tag{25}$$

and the JADE algorithm to identify the kernels corresponding to subsystems of order J_r as columns of the matrix $\mathbf{G}_{J_r,J_r}$.

– **End of Kernel Identification**

C. **Volterra Systems Equalization**

– Step 1: Based on theorem 3, compute the matrices $\mathbf{\Psi}_{J_r,J_m}$, for all $m, 1 \leq m \leq r-1$.

– Step 2: Set $l = J_r$ and compute the matrix

$$\mathbf{E}_{J_r} - \sum_{m=1}^{r-1} \mathbf{\Psi}_{J_r,J_m} = \mathbf{\Psi}_{J_r,J_r} \tag{26}$$

– Step 3: Use the first row of the computed matrix $\mathbf{\Psi}_{J_r,J_r}$ to equalize the input vector. Due to assumption A3 and the definition of $\mathbf{\Psi}_{J_r,J_r}$, the first row of $\mathbf{\Psi}_{J_r,J_r}$ equals the vector $[s(k) \quad s(k+1) \cdots \quad s(k+Q-1)]$.

– Step 4: Use the next block of Q received symbols, to compute for all $i, 1 \leq i \leq r$ the matrices $\mathbf{E}_{J_i}$. Store the matrices.

– Step 5:Repeat steps $1-4$ to equalize next block of Q symbols. Do so, until all received symbols have been processed.

– Step 6:**End of Equalization**

For Volterra systems, after equalization has been performed, the vector of the linear kernel taps can be evaluated as the first column of $\mathbf{\Psi}_{J_r,J_r}$ divided by $s(k)$. Furthermore, in order to equalize input symbols using equation(26) we only need to know the values of the first row of each of the matrices $\mathbf{\Psi}_{J_r,J_m}$.

4 Conclusions

4.1 Order Determination

The performance of the proposed algorithm in the presence of noise relies heavily on the determination of the effective rank of a matrix perturbed by noise, is the most sensitive. In [5] various criteria are given that estimate the effective rank of a perturbed matrix using its singular values. All but one out of these, use threshold values that either do not appear to be based on any explicit analytical expressions but are selected on an ad hoc basis, or lower and upper bounds for them can be derived analytically assuming known noise statistics.

The only criterion among those presented in [5] that does not use either an ad hoc threshold value or a threshold with upper and lower bounds based on known noise statistics, determines the numerical rank t of a perturbed matrix $\mathbf{B}$ from its singular values $\beta_1 \geq \beta_2 \geq \cdots \geq \beta_r$ as the index t for which $\beta_t >> \beta_{t+1}$. It is straightforward to see that it is equivalent to the criterion used by the proposed method.

Simulation experiments showed the following:

- For MIMO systems, the validity of the method was established even for low SNR values. On the other hand, the rank pattern suggested by Theorem 3 is harder to detect in the presence of noise.
- For SIMO systems the proposed method outperforms the algorithm given in [4] in all cases at least by 6dB, while MDL achieves the 90% success rate at about the same SNR levels with the proposed method. The new method outperforms the J-LSS algorithm by at least 14dB, while J-LSS performed poorly in the case of true microwave channels.
- Denoising improved dramatically the algorithm's performance for MIMO systems. For SIMO systems, denoising had a mixed effect. However, we should mention the performance boost that was achieved when denoising was applied at the J-LSS method.
- In all our simulation experiments w was chosen equal to L the upper bound of system's orders. This is because, as noticed in [8], for a fixed data length, large values of w imply fewer columns in the algorithm's data matrices that correspond to smaller sample size when projections on subspaces are evaluated. Therefore, we tried to keep w as small as possible.
- The algorithm gives analogous results for order detection of Volterra Systems. It is important to notice that using 1500 symbols the algorithm achieves high rates of successful order detection even for low SNR values such as 10dB. This is due to the special structure of the Volterra systems, that is due to linearization of the system, where the new inputs were redefined as subproducts of delayed input values.
- The computational complexity of the proposed algorithm is $O(M^3Q^2(L+1)^4+M^3QL^3(L+1)+M^3Q^2L^3(L+1)+M^3Q(L+1)^3(L-J_1+1))$ for MIMO systems. This reduces to $O(M^3Q^2(L+1)^4+M^3QL^3(L+1)+M^3Q^2L^3(L+1))$ for SIMO systems. In the above M is the number of outputs and Q is the number of output data samples used. This suggests that the oversampling rate or the number of diverse sensors M used at the receiver should be kept small to avoid heavy computations. In addition, the less output samples used and the tighter the estimate of the system orders' upper bound L is, the less expensive computations become.

4.2 LTI FIR MIMO Kernel Identification and SIMO Volterra Systems Equalization

For LTI FIR MIMO systems the algorithm allows kernel identification in steps, based on the JADE algorithm. Kernels are identified in groups, depending on the

memory length of the subsystems they belong to. Performance improvement as compared with the algorithms given in [9, 10] emanates from the fact that kernel identification is performed in steps, starting with the subsystems attaining the lower memory and progressing to the subsystems with the higher memory. The computational complexity of the proposed method is $O(Q^4 + (ML\ Q \log_2(Q)))$, where Q is the size of symbols block, This is because, even though the method is SOS based, the involvement of the JADE algorithm increases the complexity, as it requires the computation of fourth-order cumulants.

For Volterra systems, the performance of the algorithm is quite satisfactory, as compared with the ones given in [6, 11]. The overall algorithm complexity is therefore, $(\frac{N}{Q})O(ML\ Q \log_2(Q))$.

For both LTI FIR MIMO systems and SIMO Volterra systems, selecting the appropriate size Q of symbol blocks affects the correct order determination and by that the matrices E_l based on which identification/equalization are performed. Selection of Q depends on SNR as well as the complexity of the system examined, that is the number of subsystems it comprises of and their orders. Extensive simulation experiments, involving both LTI FIR MIMO and Volterra systems triggered by different types of PSK input, suggested that $Q \geq 1500$.

4.3 Future Work

As it has already been mentioned the algorithm's complexity is $O(M^3Q^2(L+1)^4 + M^3QL^3(L+1) + M^3Q^2L^3(L+1) + M^3Q(L+1)^3(L-J_1+1))$ for the order identification part of MIMO systems and $O(M^3Q^2(L+1)^4 + M^3QL^3(L+1) + M^3Q^2L^3(L+1))$ for SIMO systems. Moreover, we pay $O(Q^4 + (ML\ Q \log_2(Q)))$ operations for identification or $(\frac{N}{Q})O(ML\ Q \log_2(Q))$ for Volterra systems equalization. It is obvious that there should be some future work to improve computational complexity. This can be achieved either by ausing a recursive implementation scheme or by introducing other type of BSS techniques, with less complexity than the JADE algorithm.

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