

# Propagation limitations in all-optical networks due to nonlinear effects

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**Abstract.** The subject of this PhD thesis is the study of the impact of the nonlinearity in the performance of an optical telecom system. In particular the FWM phenomenon which is detrimental in WDM systems is analyzed theoretically and numerically, while various methods are proposed for its compensation. Finally the stability of nonlinear soliton pulses in CROWs is proposed in order to realize compact optical delay lines.

**Keywords:** Chirp, Four-Wave Mixing, modulation formats, nonlinear optics, photonic crystals, solitons, wavelength-division multiplexing (WDM).

## Introduction

In this thesis, propagation limitations in all-optical networks due to nonlinear effects and especially Four-Wave Mixing (FWM) [1]-[3] are studied. In the first part of this thesis an accurate analysis of the statistical nature of the FWM noise was carried out using Monte Carlo (MC) and Multicanonical MC (MCMC) simulations. Such an analysis is of great importance in network design and modeling since FWM is an important source of noise in dense wavelength-division multiplexing (DWDM) networks that employ nonzero-dispersion fibers (NZDF). In these systems, FWM causes the dominant nonlinear distortion, especially if high input powers, narrow channel spacing and dispersion compensation are used. The MCMC method was also applied to study the impact of IP traffic burstiness on the performance of an IP over MPLS-based DWDM network limited by FWM and In-band crosstalk.

In the second part of this thesis two new methods based on a hybrid amplitude-/frequency-shift keying (ASK/FSK) modulation and pulse prechirping are proposed for the suppression of the FWM effect. The next part addresses the issue of the choice of optical modulation format in a multispan WDM system using G.655 fiber. Various modulation formats are assessed by numerical simulation in the presence of fiber nonlinearities with FWM being the dominant effect. Finally, in the last part of this thesis, the propagation of optical slow light solitons in Coupled

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Resonator Optical Waveguides (CROWs) is proposed as an alternative optical delay line design method for reducing dispersion effects.

### Statistical nature of the FWM noise

In this section, the MCMC method was applied for the study of the statistical behavior of the FWM noise in a WDM network. At the receiver, the photocurrent is proportional to the optical power and hence to  $|E|^2$  where  $E=E^{(m)}$  or  $E=E^{(s)}$ . In practical applications, it can be assumed that  $\Delta\beta \gg a$  and  $\exp(-aL) \ll 1$ , where  $a$ ,  $L$  and  $\Delta\beta$  are the optical losses, the length of the optical link and the phase match factor [3] of the FWM process respectively. In this case the photocurrent at the detector is written as:

$$S^{(m)} = k|E^{(m)}|^2 \approx kP_n e^{-aL} + 2k\delta\sqrt{P_n e^{-aL}} I_m \quad (1a)$$

$$S^{(s)} = k|E^{(s)}|^2 \approx k\delta^2 I_s \quad (1b)$$

where  $k$  is the receiver responsivity,  $P_n$  is the input peak power of the channel  $n$  and

$$\delta = \frac{\gamma c}{2\pi\lambda^2 D \Delta f^2} P_n^{3/2} e^{-aL/2} \quad (2a)$$

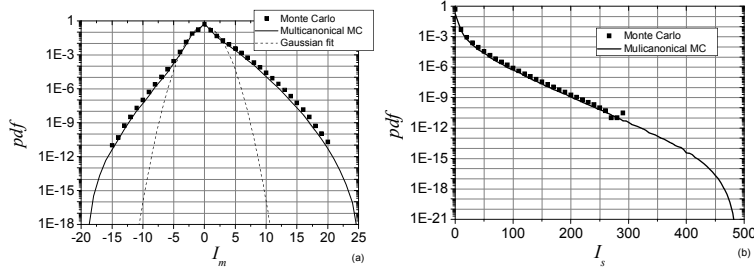
$$I_m = \frac{1}{3} \sum_{pqr} B_p B_q B_r \frac{d_{pqr}}{|p-n||q-n|} \cos(\theta_{pqr} - \theta_n) \quad (2b)$$

$$I_s = \left( \frac{1}{3} \sum_{\substack{pqr \\ r \neq n}} B_p B_q B_r \frac{d_{pqr}}{|p-n||q-n|} \cos \theta_{pqr} \right)^2 + \left( \frac{1}{3} \sum_{\substack{pqr \\ r \neq n}} B_p B_q B_r \frac{d_{pqr}}{|p-n||q-n|} \sin \theta_{pqr} \right)^2 \quad (2c)$$

where  $\gamma$  is the nonlinear coefficient,  $D$  is the fiber chromatic dispersion coefficient,  $\lambda$  is the wavelength of the signal,  $c$  is the speed of light in vacuum,  $\Delta f$  is the channel spacing,  $d_{pqr}$  is the degeneracy factor ( $d_{pqr}=3$  when  $p=q$ ,  $d_{pqr}=6$  when  $p \neq q$ ) and  $\theta_n$  is the input phase in the mark state, respectively, of the given channel  $n$ .

Equations (1a) and (1b) provide an expression for the photocurrent in the mark and space state in terms of two new variables  $I_m$  and  $I_s$  given by (2b) and (2c) respectively. It is interesting to note that for a given number of channels, these new variables depend only on the bits and the phases of the optical signals. Once the pdf of  $I_m$  and  $I_s$  is determined, the pdf of  $S^{(m)}$  and  $S^{(s)}$  can also be determined, using the theorem of transformation of random variables.

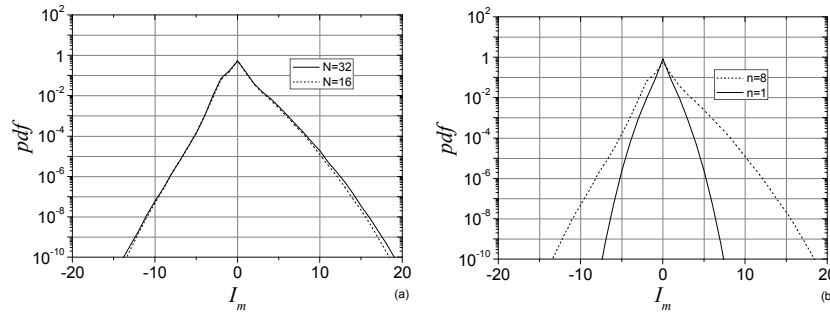
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**Fig. 1.** The PDFs of a)  $I_m$  and b)  $I_s$  for  $N=16$  calculated using the MCMC (solid lines) and conventional MC (dots) methods. Also shown with dotted lines in (a) is a Gaussian distribution with the same standard deviation as  $I_m$ .

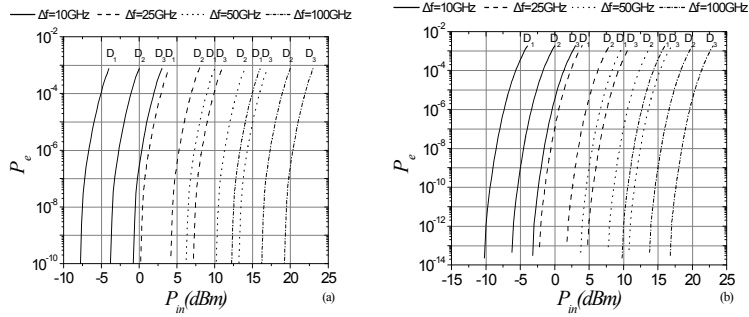
The MCMC method [4] provides a simple, accurate and efficient tool for the computation of the PDF of the FWM noise (figure 1) and overcomes the limitations of the conventional MC method. The optical phases of all channels are assumed to be uniformly distributed within  $[0, 2\pi]$ , due to phase noise, and the data bits are assumed to be in the mark and space state with equal probability,  $P(B_i=0 \text{ or } 1)=1/2$ . It is shown that the pdf in the mark state is not symmetric as assumed in previous studies [1], [2].

The MCMC method is also far more accurate than the Gaussian model (figure 1a) because it takes into account the correlation of the FWM noise components. Using the MCMC method it was also shown that the PDF converges quickly to its asymptotic form as the number of channels  $N$  increases (figure 2a). Indeed for  $N>32$ , the change in the PDF is hardly distinguishable even in a log-scale.



**Fig. 2.** a) The PDFs of  $I_m$  for the central channel in the case where  $N=16$  (dashed lines) and  $N=32$  (solid lines) and b) The PDF of  $I_m$  for  $N=16$ ,  $n=8$  (dashed lines) and  $N=16$ ,  $n=1$  (solid lines).

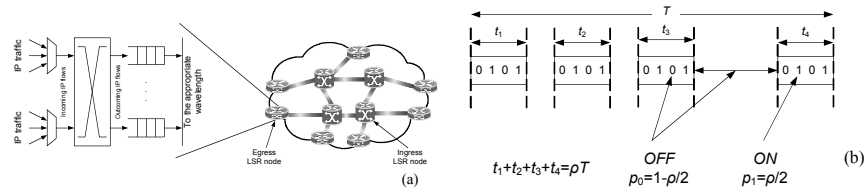
These results can prove useful for a system designer who wants to estimate the implications of the FWM-induced distortion in a WDM network. In particular, the error probability  $P_e$  can be computed from the pdfs  $S_m$  and  $S_s$  using numerical integration (figure 3).



**Fig. 3.** BER as a function of the input peak power  $P_{in}$  in the mark state, for a)  $N=8$  and b)  $N=16$ . The values of the chromatic dispersion used are  $D_1 = 2 \text{ ps / nm / Km}$  and  $D_2 = 5 \text{ ps / nm / Km}$

### Impact of IP traffic Burstiness

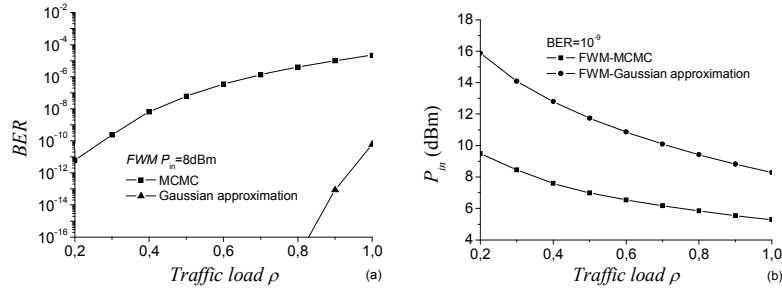
The performance of Wavelength Division Multiplexing (WDM) optical networks can be severely degraded due to the existence of signal dependent noises such as the Four-Wave Mixing noise whose statistical behavior will depend on the statistics of the signal. To assess the implication of such noises, the bits are usually assumed to be in the mark and space state with equal probability. This approach however, does not take into account the effect of traffic burstiness in packet switched networks. In this section, the influence of traffic burstiness in the performance of the network is numerically analyzed in the case of an IP over WDM network. The network employs the MultiProtocol lambda Switching (MP $\lambda$ S) scheme [5] (figure 4a).



**Fig. 4.** a) The Label Edge Router of an IP over MP $\lambda$ S-based DWDM network and b) “1” and “0” generated under bursty traffic

At the ingress nodes, the packets are forwarded according to their wavelength. In order to take into account the traffic burstiness, each wavelength is modeled as an M/G/1 system. In this case the probabilities of “1” and “0” are  $p_0=1-\rho/2$  and  $p_1=\rho/2$  [6] where  $\rho$  is the traffic load (fig. 4b). The dependence of  $p_1$  on  $\rho$  is a first indication of the influence of traffic burstiness in the system performance: the decision variable in a FWM limited system depends on the bit statistics and hence on  $\rho$ .

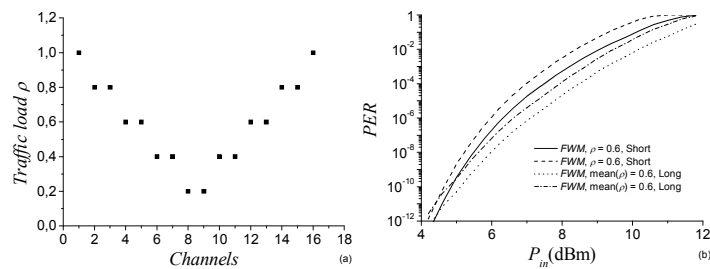
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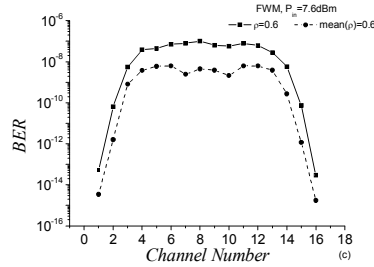


**Fig. 5.** Comparison between the MCMC method and the Gaussian approximation. a) BER vs traffic load  $\rho$  for  $P_{in}=8\text{dBm}$  and  $\text{SXR}=12\text{dB}$  and b)  $P_{in}$  and SXR for obtaining  $\text{BER}=10^{-9}$ .

The numerical analysis is carried out using the MCMC method. It is deduced that the network performance is intimately related to the traffic load (figure 5a). The Gaussian approximation based on the Central Limit Theorem cannot lead to accurate results in the presence of the FWM noise since the decision variable  $D$  can not be written as a sum of independent random variables. To further understand the implications of the error in the Gaussian approximation, the required values of the input power corresponding to a BER equal to  $10^{-9}$ , are plotted in fig. 5b for various  $\rho$ .

It was previously shown that the FWM degradation was more severe for the central channels than the edge channels of a WDM system. It is then obvious that one way to reduce the FWM noise and increase the transmission power is to redistribute the traffic so that the edge channels carry heavier traffic than the central ones. Figure 6 illustrates that careful traffic engineering can improve the system performance in terms of the BER by at least one order of magnitude. These results imply that, when analyzing the performance of the network, traffic burstiness is an important issue that must be taken into account.





**Fig. 6.** a) Traffic load distribution along the channels, b) PER as a function of the input peak power  $P_{in}$  when all the channels are equally loaded with  $\rho=0.6$  (solid line) and unequally loaded with mean  $\rho$  equal to 0.6 (dashed line) and c) BER values for all channels at  $P_{in}=7.6\text{dBm}$ .

## New Techniques for the Suppression of the FWM-Induced Distortion in NZD Fiber WDM Systems

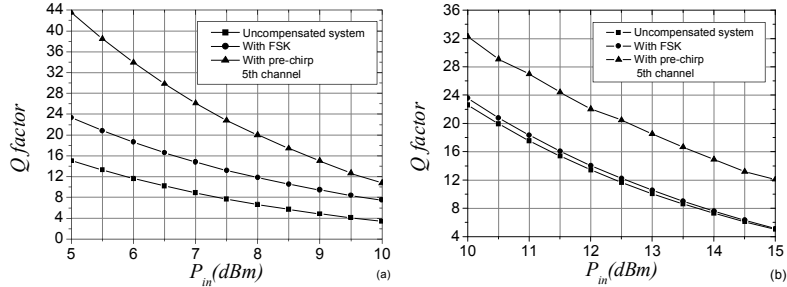
In order to reduce FWM-induced distortion two new techniques, the hybrid ASK/FSK modulation and the use of pre-chirped pulses, are proposed. In a WDM system with equally spaced channels, the central frequency of the products will coincide with some of the central frequencies of the channels. In order to reduce the number of FWM products that coincide with the WDM channels, one solution is to modulate the WDM signals using a special kind of FSK modulation. In the context of this special scheme, the WDM channels are divided into pairs and on each pair the channels follow the same FSK modulation.

Another solution for the reduction of the effect of the FWM induced distortion is optical pre-chirping. Since the efficiency of the FWM products are inversely proportional to the phase mismatch, it follows that reducing the phase coherence may reduce the power of the FWM noise. One way to reduce this coherence is through pulse pre-chirping. There are several methods to produce a pre-chirped signal such as cascading intensity and phase modulators or using dispersion-compensating devices like chirped fiber gratings and DCFs. In this work, the latter technique was chosen due to its ease of implementation.

It is shown that both techniques can greatly improve the  $Q$ -factor in a 10Gb/s WDM system (figure 7a). This happens even for very high input powers ( $\sim 10\text{dBm}$ ), where the degradation of the conventional WDM system is prohibitively high.

The proposed methods are also applied and tested in higher bit rates - 40Gbps (figure 7b). It is deduced, that although the hybrid ASK/FSK modulation technique marginally improves the system performance, the optical pre-chirp technique can still be used to greatly increase the maximum allowable input power of the system.

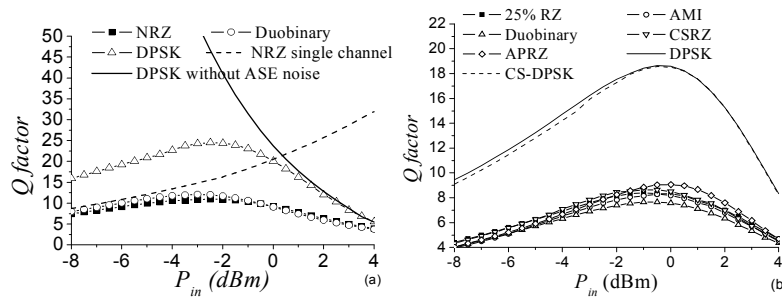
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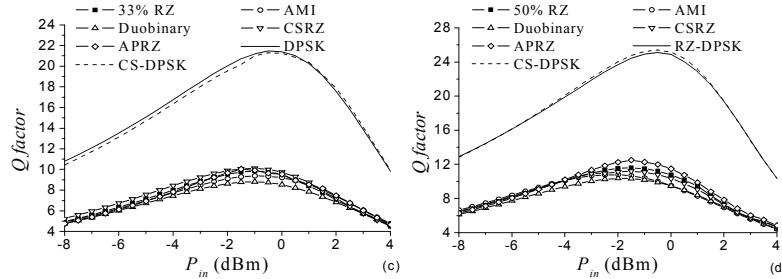


**Fig. 7.** Q factor of the central channel as a function of the input power  $P_{in}$  for an a) 10Gbps system with 8 channels and 50GHz channel spacing and b) 40Gbps system with 8 channels and 200GHz channel spacing

### Non-linearity Tolerance of Optical Modulation Formats in NZD Fibers

This section addresses the issue of the choice of optical modulation format in a multi-span WDM system using G.655 fiber. Various modulation formats [7] are assessed by numerical simulation in the presence of fiber non-linearities with Four Wave Mixing (FWM) being the dominant effect. FWM noise has complex statistical behavior and thus the impact of the modulation format cannot be readily understood. Phase modulation formats are also degraded by the Gordon-Mollenauer (G-M) effect, which is due to the conversion of amplitude fluctuations created by the Amplified Spontaneous Emission (ASE) noise to phase fluctuations through Kerr non-linearity. Hence, one must resort to numerical simulations to accurately take into account the impact of fiber non-linearity. It is shown that the various amplitude modulation alternatives result in more or less the same performance. Phase modulation schemes such as DPSK [8], [9] drastically increase the system performance leading to an increase of the Q-factor by almost 3dB and of the optimum power by 1dB (figure 8).

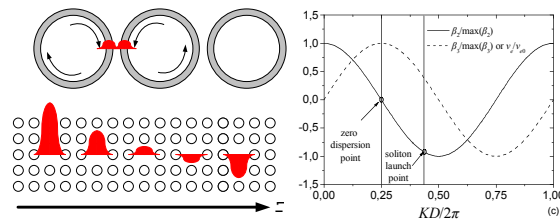




**Fig. 8.**  $Q$  factor as a function of input peak power for a) NRZ pulses, b) 25% RZ pulses, c) 33% RZ pulses and d) 50% RZ pulses

### Optical Delay Lines Based on Coupled Resonator Optical Waveguide Soliton Propagation

Soliton Pulses are investigated as a means to mitigate dispersion-induced broadening and further reducing the group velocity in CROWs. High index contrast devices based on photonic wires or photonic crystals may enable further miniaturization and increased optical chip functionality. One of the main challenges in the field, is the realization of compact all-optical delay lines. Slow light structures such as coupled resonator optical waveguides (CROWs) [10] could possibly provide an attractive means of achieving large optical delays on a chip scale. As light slows down, the nonlinear properties are enhanced and this could also find applications in optical signal processing. Second and higher order dispersion however currently limits the performance of such devices [11].



**Fig. 9.** Linear and nonlinear (soliton) propagation in CROWs

Figure 9 illustrates the group velocity second and third order dispersion coefficients  $v_g$ ,  $\beta_2$  and of a microring or photonic crystal defect CROW, where  $K$  is the propagation constant and  $D$  is the distance between the centers of two consecutive resonators. At  $KD=\pi/2$ , one obtains  $\beta_2=0$  and hence second order dispersion does not cause pulse broadening. This is the point usually considered when the CROW is operated in the linear regime. Unfortunately however, the group velocity is maximum there implying small achievable delays and the signal can still be distorted by third



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order dispersion. Although residual third order dispersion can in principle be compensated, the device length can still be long.

On the other hand, one may think of launching the pulse near  $KD=\pi$ , where the group velocity is minimum and  $\beta_3=0$ , while the second order dispersion can be compensated using the Kerr-induced, Self Phase Modulation (SPM) using sech soliton pulses. At this regime, the residual soliton broadening is only due to the fourth order dispersion  $\beta_4$  and is much smaller than that experienced near the zero-dispersion point. These arguments point out that soliton pulses propagating at the nonlinear regime would experience much lower pulse broadening than linear pulses launched at the zero dispersion point. To ascertain whether CROW soliton propagation could indeed have a practical bearing on optical delay line design, one must evaluate the required pulse peak power and estimate the amount of optical losses that can be tolerated in the system. To describe optical propagation in CROWs, one uses the propagation equation:

$$j\left(\frac{\partial\Phi}{\partial t} + v_e \frac{\partial\Phi}{\partial z} + \frac{a}{2}\Phi\right) + \sum_{l \geq 2} j^{m(l)} \frac{\beta_l}{l!} \frac{\partial^l \Phi}{\partial z^l} + \gamma |\Phi|^2 \Phi = 0 \quad (3)$$

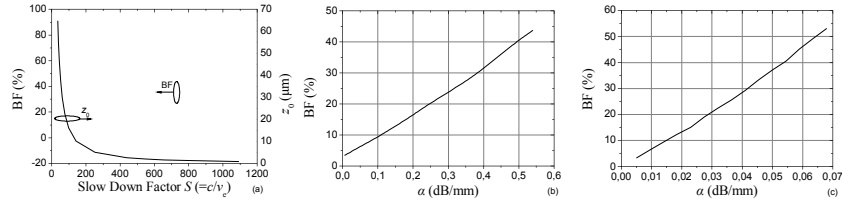
where  $\Phi$  is the optical pulse envelope,  $v_e$  is the group velocity,  $a$  is the optical loss coefficient,  $\beta_l$  is the  $l^{\text{th}}$  order dispersion coefficient and  $\gamma$  is the SPM coefficient of the CROW, while  $m(l)=\text{mod}(l,2)$ . The values of these coefficients in terms of the CROW modal fields can be found in [12]. The initial soliton pulse is given by  $\Phi_0 \text{sech}(z/z_0)$ , where  $z_0$  is the soliton spatial width, related to the soliton duration  $t_0$  by  $t_0=z_0/v_e$  while  $\Phi_0$  is determined by  $\Phi_0=|\beta_2|/\gamma z_0^2$ .

To calculate the power, one uses the well known Poynting theorem formula,

$$P(z) = \langle \Pi(z) \rangle_t \cong \left\langle \int (\mathbf{E} \times \mathbf{H}) \cdot \hat{z} dS \right\rangle_t \quad (4)$$

where  $\mathbf{E}$  and  $\mathbf{H}$  are the electric and magnetic field. The fields inside the CROW which can be calculated from the modes of the cavities and the envelope  $\Phi(z,t)$  based on the tight binding approximation as outlined in [12]. The modes of the cavities can be calculated using the Plane Wave Expansion (PWE) method.

To examine the performance of the CROW delay line in the presence of higher order dispersion, one can numerically solve (4) including higher order dispersion terms (up to  $l=6$ ). Figure 10a, depicts the broadening factor for the case of 2 rod spacing. As shown in figure 10a, in the case of two rod spacing between the cavities, the broadening exceeds 30% for A Slow Down factor  $S=c/v_e=800$  in which case,  $R=v_e/v_{e0}=0.01$ . Note that a broadening of 30% roughly corresponds to a power penalty of 1dB. This implies that the nonlinear CROW can achieve about 100 times smaller delays than a linear CROW operated at resonance ( $v_e=v_{e0}$ ). Note that  $S=c/v_e=800$  implies that a 10ns delay (required to store 100 bits at 10Gb/s can be accommodated in just 3.8mm of propagation length.



**Fig. 10.** a) Variation of  $BF$  and  $z_0$  with respect to  $S$ , b-c) Variation of  $BF$  with respect to  $\alpha$

The influence of optical loss is depicted in figure 10b and 10c where the broadening factor is plotted as a function of the loss coefficient for the case of a 1ns and 10ns delay respectively. In both cases the spacing between two successive cavities is two rods and  $R=0.1$ . The figure suggests that soliton pulses are sensitive to loss. For example, in the case of 10ns delay, increasing the loss from  $\alpha=0.03$  dB/mm to  $\alpha=0.04$  dB/mm leads to a 10% increase of the pulse width. A 30% broadening is obtained for  $\alpha=0.042$  dB/mm and this indicates that the required loss value is very low, even compared to the present state-of-the-art loss values for ring resonator CROWS. These considerations demonstrate the importance of reducing the optical losses and towards this end, various distributed amplification schemes can be used such as Raman amplification or quantum wells. Efficient PC slab designs can also lead to optical loss reduction. For smaller delays, the required losses are somewhat relaxed and a 30% broadening is obtained for  $\alpha=0.383$  dB/mm in the case of a 1ns specified delay.

## Conclusions

In this thesis, the MCMC method has been applied for a study of the statistical behavior of FWM noise in a WDM network. The obtained results were used in estimating the performance of a system assuming continuous or bursty traffic models. The MCMC method was proved far more accurate than the Gaussian model because it takes into account the correlation of the FWM noise components. It was also illustrated that careful traffic engineering can improve the system performance in terms of the BER by at least one order of magnitude.

Two techniques—hybrid ASK/FSK modulation and prechirping the optical pulses—were proposed to suppress the FWM-induced distortion which can pose important limitations on the input power of a WDM system. It was proved that both techniques greatly improve the performance of a WDM system.

A performance comparison of modulation formats for a multispan WDM system with G.655 fibers was presented. It was shown that advanced amplitude formats do not result in increased performance while phase modulation schemes (DPSK and CSDPSK) are more advantageous.

Finally, the performance of a soliton-based CROW delay line where nonlinearity is used to compensate for the second order dispersion-induced pulse broadening was analyzed. A propagation equation describing the soliton propagation under the influence of higher order linear effects was given. Design equations were provided in

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order to calculate the peak power of the soliton and to choose the soliton width in order to eliminate soliton attraction and collision. The importance of higher order nonlinear effects was also investigated. It was shown that if the optical losses are kept low, the soliton-based CROW delay line can achieve nanosecond delay at a propagation length of a few millimeters due to the high slow down factors obtained. The soliton-based delay line could, therefore, provide a path towards realizing integrated optical buffers.

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