Study of the effect of channel parameter estimation errors on telecommunication systems performance

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Abstract. This work investigates how channel parameter estimation errors affect the performance of telecommunication systems. The telecommunication channel is (usually) modeled as a linear filter which is comprised of a fixed number of coefficients. In this sense, the term "channel parameters" is used to denote either the *total number* of these coefficients or the *exact value* of each one of them. Numerous algorithms/procedures in the telecommunication literature assume *exact* knowledge of these parameters and subsequently build upon this knowledge to carry out a specific telecommunication task. In real world conditions, however, nothing is known *a priori* and all such quantities must be *estimated* by means of a suitable procedure. Nevertheless, any estimation procedure is subject to errors, which can cause severe degradation of the overall system performance. The present study unfolds along two main axes: The performance analysis (in terms of equalization quality) of a blind second order statistics (SOS) equalization algorithm when *channel order* (i.e. number of channel coefficients) is *unknown* and the performance analysis (in terms of sum-rate degradation) of the uplink of a multiuser multiantenna system, when the *exact* values of channel coefficients for each user are *unknown*. Using rigorous mathematical analysis based on perturbation theory results together with simulation evidence, conditions and conclusions are derived as to the instances where estimation errors lead (or do not lead) to a significant system performance deterioration.

1 Introduction

A common trait of many telecommunication-related algorithms/procedures is the assumption that various aspects of the telecommunication system are known *a priori*. This (assumed) knowledge is the cornerstone upon which the algorithm builds to complete the task it was designed for. In a practical scenario, though, nothing is known a priori and all required knowledge must be gathered through estimation. However, estimation is by nature prone to errors that make the ground upon which the algorithms operate more or less shaky. Thus, the ability

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to quantify the effect of such errors on overall system performance is an issue of paramount practical importance.

Performance analyses made to this end aim to unveil those algorithms whose output will degrade by "just a little" in the presence of "small" estimation errors. These algorithms will be called *robust*. On the other hand, *non-robust* algorithms will collapse even in the presence of "small" estimation errors. Although both types of algorithms are theoretically sound, only the robust ones are expected to be of any practical use.

In the field of telecommunications, assumptions about knowledge of channel parameters are common. In an abstract formulation, the telecommunication channel is frequently modeled as an FIR (Finite Impulse Response) filter with a fixed number of coefficients. Thus the term *channel parameters* either denotes the coefficients' *total number* or *the exact value* of each coefficient, the specific meaning being determined from context. Knowledge of channel parameters is exploited by *equalization* algorithms to construct equalizers, i.e. devices that strive to compensate the ubiquitous telecommunication impairment of Intersymbol Interference (ISI). In addition, channel coefficient knowledge is necessary when the calculation of *capacity* is required, i.e. the maximum rate at which information can be transmitted through a given system.

In order to precisely quantify the deterioration in performance due to estimation errors, extensive use of *perturbation theory* results has been made in the framework of the present study. Broadly speaking, the topic of perturbation theory is the study of the variations of a function as its arguments get perturbed, i.e. changed by a "little" (the exact meaning of "little" is largely case dependent). This is a very general definition that encompasses numerous problems. For example if A^{-1} is the inverse of matrix A and is seen as a function of the elements of A, then it is the duty of perturbation theory to determine how A^{-1} changes when A is perturbed by E. In other words if $\widetilde{A} = A + E$ how close would $\widetilde{A}^{-1}$ and A^{-1} be?

The application of perturbation theory in assessing the effect of estimation errors in telecommunication systems is a two step process: First an *ideal* case is conceived. The ideal case is simply the case where exact knowledge of every required parameter is assumed. This is clearly non-realistic and hence the name *ideal*. The algorithm at hand copes well with its prescribed task in the ideal case. The second step consists of taking into account the estimation inaccuracies. This case is naturally named "realistic". Algorithms may face serious problems due to the inaccuracies in the realistic case. The role of perturbation theory is to relate the realistic with the ideal case and assess how close to each other are, under the assumption that the magnitude of the perturbation (i.e. the estimation errors) is "small" (like earlier, the exact meaning of "small" is case dependent). The outcome of this effort should reveal whether the algorithm at hand is robust or not, as previously discussed.

Based on the arsenal of pertubation theory, the first part of the study focuses on the equalization of SIMO (Single Input Multiple Output) systems. SIMO systems are modeled as parallel channels (also known as subchannels) that are driven by a common input and arise in cases where receivers oversample the channel output at a rate that is an integer multiple of the transmission symbol rate (also known as *baud* rate) and/or are equiped with multiple receive antennas. The role of the equalizer is to appropriately process the outputs of the parallel channels in order to estimate in the best possible way their common input. The equalizers originally proposed in [1] are investigated. These equalizers have the following properties:

- They minimize the mean square error (MMSE) between transmitted and estimated symbols.
- The equalizers are constructed *without* relying on the help of *training sequences*. Thus, they belong in the so called *blind equalization* regime.
- Their construction is based *solely* on channel output second order statistics (SOS), i.e. correlation, covariance matrices.

The equalizer construction method described in [1] assumes *a priori* knowledge of *channel order*, i.e. it requires that the *number* of taps (coefficients) of each subchannel be known *in advance*. Of course in a real world environment such knowledge can only be acquired through estimation. In the case of microwave links, subchannels are comprised of a few "large" taps while all others are much smaller. In such a context, channel order estimation can be tricky. This study thoroughly assesses the effect of the *estimated channel order* on *equalization quality*. The analysis along with its results and conclusions is reviewed in section 2.

The second leg of the study focuses on *capacity* issues of telecommunication systems. In particular the uplink of a wireless, multiuser, multiantenna system is examined. The scenario assumes many users who want to communicate simultaneously with a common base station (also known as *uplink* communication). Both the base station and users are equipped with multiple antennas as it has been proven in [2] that this choice leads to systems with increased capacity. The use of multiple antennas raises the issue of optimal transmission signal design. The transmitted signals must be designed in such a way that the sum-rate (i.e. the collective rate of all users) is maximized, therefore achieving system capacity.

The necessary condition of optimality is described in [3]. Under the assumptions that channels of all users change slowly (slow fades) so that they can be considered fixed for the duration of a transmission block, are i.i.d. distributed and the number of users is much greater than the number of antennas at the base station then *almost all* users should employ *beamforming* to achieve system capacity.

Beamforming is a simple transmission technique by which the transmitted signal is of the form $\mathbf{x}[n] = s[n] \times \mathbf{w}$, where $s[n]$ denotes the transmitted symbol at discrete time instant n and $\mathbf{w}$ is a weight vector, whose i-th component pertains to the i-th transmission antenna. Vector $\mathbf{w}$ is known as *beam vector*.

If *all users* employ beamforming (instead of *almost all*) then it can be observed by simulation ([3]) that the sum-rate at the uplink *approximates* system capacity. This is the kick-off point of this study. Each user makes use of a different beam vector, whose computation is based on the *knowledge of the respective* *channel*. As thoroughly explained earlier this knowledge can only arise from estimation procedures. This study aims at (and succeeds in) computing the mean *sum-rate reduction* due to estimation errors in the channels of users as a function of channel estimation error covariance matrix. Due to the short nature of the present paper, this second leg of the dissertation will not be further analyzed.

2 Blind, second order statistics (SOS) equalization

2.1 Channel model and basics

Let a discrete time system consist of p parallel, linear, FIR filters (subchannels), each one of order M (i.e. $M+1$ taps long) driven by a common input. At the output of each subchannel white noise of equal power is added. This is a generic SIMO model[1]. Let $s(n)$ denote the *scalar* common input to the system at discrete time n and $\mathbf{x}(n)$ the p-component *vector* denoting the output at the same instant. The input-output relation is given by the convolution: $\mathbf{x}(n) = \sum_{i=0}^{M} \mathbf{h}_i s(n-i)$, where $\mathbf{h}_i$ is the p-component vector grouping the i-th coefficient of each subchannel. By stacking the $L+1$ most recent outputs, vector $\mathbf{X}_L(n) = [\mathbf{x}(n)^T \cdots \mathbf{x}(n-L)^T]^T$ can be constructed. This vector is compactly expressed as:

$$\mathbf{X}_L(n) = \mathcal{T}_L(\mathbf{H}_M)\mathbf{s}_{L+M}(n) + \mathbf{w}_L(n), \tag{1}$$

by defining the $p(L+1) \times (L+M+1)$ generalized Sylvester matrix (aka "filtering matrix")

$$\mathcal{T}_L(\mathbf{H}_M) = \begin{bmatrix} \mathbf{h}_0 \cdots\cdots \mathbf{h}_M & & \\ & \ddots & \ddots & \\ & & \mathbf{h}_0 \cdots \cdots \mathbf{h}_M \end{bmatrix},$$

vector $\mathbf{s}_{L+M}(n) = [s(n) \cdots s(n-L-M)]^T$ and vector $\mathbf{w}_L(n) = [\mathbf{w}(n)^T \cdots \mathbf{w}(n-L)^T]^T$, where $\mathbf{w}(n)$ is the p-element vector grouping the noise samples at time instant n. Vector $\mathbf{H}_M = [\mathbf{h}(0)^T \cdots \mathbf{h}(M)^T]^T$ groups the taps of all subchannels.

A (linear) *equalizer* is a set of p linear, FIR filters of order L each, connected serially to the SIMO system. As a result, the constituent filters of the equalizer are arranged in a MISO (Multiple Input Single Output) setting. All p filter outputs in the set are added to yield the *single* equalized output $z(n)$. $z(n)$ should approximate the input $s(n)$ (or a delayed version thereof) in some sense. The mode where equalizer coefficients are chosen so that $E\{\|z(n) - s(n-i)\|_2^2\}$ is minimized is known as *minimum mean square equalization (MMSE)* equalization. If equalizer coefficients are properly placed in vector $\mathbf{g}_{L,i}$ then the equalized output $z(n)$ can be expressed as $z(n) = \mathbf{g}_{L,i}^H \mathbf{X}_L(n)$. The desired $\mathbf{g}_{L,i}$ can be computed by solving the Wiener-Hopf equation:

$$E\{\mathbf{X}_L(n)\,\mathbf{X}_L^H(n)\}\,\mathbf{g}_{L,i} = E\{\mathbf{X}_L(n)\,s_{n-i}^*\} \tag{2}$$

[1] All model variables are assumed complex.

The left-hand side (LHS) of (2) depends only on system output and can therefore be computed blindly, i.e. *without* using training sequences. The right-hand side (RHS), however, depends on the input through s_{n-i} and its computation would *traditionally* mandate the use of such sequences. In [1] however, a novel method is introduced that computes $E\{\mathbf{X}_L(n)\, s^*_{n-i}\}$ in a *blind* fashion using SOS of the output. The method essentially amounts to manipulating autocorrelation matrices of the output in such a way that a certain matrix, symbolized as $\mathbf{\Delta D}_i$, is computed. It is then shown that $\mathbf{\Delta D}_i = \mathbf{H}_i \mathbf{H}_i^H$, so $\mathbf{H}_i$ can be retrieved[2] by EVD on $\mathbf{\Delta D}_i$.

An important remark at this point is that all SOS-based equalization methods require that $\mathcal{T}_L(\mathbf{H}_M)$ be *left invertible*, a condition that is synonymous with the equalizability of the system. This demand gives rise to the so called *zero-forcing conditions*: if $p(L+1) \geq L+M+1$ (i.e. $\mathcal{T}_L(\mathbf{H}_M)$ is "tall") *and* the subchannels *do not* have common zeros then $\mathcal{T}_L(\mathbf{H}_M)$ has full column rank, which guarantees its left invertibility. The first component of the zero-forcing conditions implies a *minimum order* L for the equalizers, namely $L \geq M/(p-1) - 1$ (in the 2-subchannel case, $L \geq M-1$). This threshold on equalizer length depends on subchannel order and number. Therefore *inaccurate knowledge* of the *subchannel order* M directly affects equalizer order L and this may lead to significant system performance degradation.

2.2 The case of two long subchannels without noise

Striving to highlight the dependence of equalization quality on accurate subchannel order knowledge, the study focuses on the noiseless case. Additive white noise naturally inhibits restoration of the transmitted sequence. In the absence of noise, the solution of (2) gives rise to equalizers that *exactly* restore the input (or a delayed version of it). By further assuming a white input sequence, it holds that[3] $E\{\mathbf{X}_L(n)\,\mathbf{X}_L^H(n)\} = \mathcal{T}_L(\mathbf{H}_M)\,\mathcal{T}_L^H(\mathbf{H}_M)$ and $E\{\mathbf{X}_L(n)\, s^*_{n-i}\} = \mathcal{T}_L(\mathbf{H}_M)(:,i+1) \triangleq \widehat{\mathbf{H}}_i$. In other words $\widehat{\mathbf{H}}_i$ is the $(i+1)$-st column of the filtering matrix $\mathcal{T}_L(\mathbf{H}_M)$, $i = 0, \ldots L+M$. Thus, equation (2) is written as:

$$\left(\mathcal{T}_L(\mathbf{H}_M)\,\mathcal{T}_L^H(\mathbf{H}_M)\right)\mathbf{g}_{L,i} = \widehat{\mathbf{H}}_i \tag{3}$$

Moreover, the study focuses on the 2-subchannel case.

In the context of microwave radio links, subchannels are usually comprised of many taps only few of which are "large" (and most often contiguous), while all others are much smaller. In such a scenario an M-th order subchannel may

[2] Actually $\mathbf{H}_i$ can be determined within a *sign ambiguity* but this is the best that can be hoped for any SOS-based blind equalization algorithm. This ambiguity, however, is a triviality of the implementation and will not be taken into account in the subsequent analysis.

[3] It is assumed that the expected values $E\{\cdot\}$ are known with infinite precision. Since the goal of the study is to investigate the dependence of equalization quality on *subchannel order estimation errors*, all other quantities (such as the aforementioned expected values) are assumed to be known *perfectly*.

be *incorrectly* estimated to be of order $L+1$ where $L+1 < M$. Zero-forcing conditions dictate the use of equalizers whose order is *at least* $M-1$. However, due to the erroneous estimation and in an effort to respect zero-forcing conditions, equalizers of minimum order L will be used in practice. As a consequence, the zero forcing conditions are *unintentionally violated* and it is interesting to know what happens when an L-th order equalizer is applied to M-th order subchannels, where $L < M-1$.

2.3 A useful partition and the definitions of ideal and realistic cases

As a first step towards analysis the following partition, first proposed by A. P. Liavas, is introduced:

$$\mathbf{H}_M = \mathbf{H}^z_{L+1} + \mathbf{D}^z_{L+1}$$

where

$$\mathbf{H}^z_{L+1} = [\underbrace{\mathbf{0}^T \cdots \mathbf{0}^T}_{m_1} \underbrace{\mathbf{h}^T_{m_1} \cdots \mathbf{h}^T_{m_2}}_{L+2} \underbrace{\mathbf{0}^T \cdots \mathbf{0}^T}_{M-m_2}]^T$$

$$\mathbf{D}^z_{L+1} = [\underbrace{\mathbf{h}^T_0 \cdots \mathbf{h}^T_{m_1-1}}_{m1} \underbrace{\mathbf{0}^T \cdots \mathbf{0}^T}_{L+2} \underbrace{\mathbf{h}^T_{m_2+1} \cdots \mathbf{h}^T_{M}}_{M-m_2}]^T$$

and $0 \le m_1 < m_2 \stackrel{\triangle}{=} m_1 + L + 1 \le M$. Here $\mathbf{H}^z_{L+1}$ groups the $L+2$ consecutive block-terms of $\mathbf{H}_M$ having the largest energy, while replacing all the rest by zeros. $\mathbf{H}^z_{L+1}$ is called the *"(L+1)-st order zero-padded significant part"* of $\mathbf{H}_M$ and $\mathbf{D}^z_{L+1}$, the complement of $\mathbf{H}^z_{L+1}$, is referred to as *"the unmodeled tails"*. Vector $\mathbf{H}_{L+1} = [\mathbf{h}^T_{m_1} \cdots \mathbf{h}^T_{m_2}]^T$ is defined to exclusively contain the significant part. Without loss of generality it is assumed that $\|\mathbf{H}_M\|_2 = 1$ and $\|\mathbf{D}^z_{L+1}\|_2 = \epsilon$ where $\epsilon \ll 1$.

The ideal case. Let the subchannels be described by $\mathbf{H}_{L+1}$ and the equalizers be of order L. This is the ideal case! The zero-forcing conditions are not violated in this scenario. In fact the *minimum length* equalizers allowed by the zero-forcing conditions are used. The equalizers for various delays are found by solving:

$$\left(\mathcal{T}_L(\mathbf{H}_{L+1})\, \mathcal{T}^H_L(\mathbf{H}_{L+1})\right) \mathbf{g}_{L,i} = \mathbf{H}_i \tag{4}$$

where $\mathbf{H}_i \stackrel{\triangle}{=} \mathcal{T}_L(\mathbf{H}_{L+1})(:, i+1)$, $i = 0, \ldots 2L+1$, i.e. $\mathbf{H}_i$ is the $(i+1)$-st column of the filtering matrix $\mathcal{T}_L(\mathbf{H}_{L+1})$.

The realistic case. The true subchannel vector $\mathbf{H}_M$ is taken into account in this case, while the equalizers are still of order L. As mentioned earlier, since $L < M-1$, the zero-forcing conditions are violated. The equalizers for varying delay values are found by solving:

$$\left(\mathcal{T}_L(\mathbf{H}_M)\, \mathcal{T}^H_L(\mathbf{H}_M)\right) \widetilde{\mathbf{g}}_{L,i} = \widetilde{\mathbf{H}}_i \tag{5}$$

The tilde over $\widetilde{\mathbf{H}}_i$ implies it is the perturbed version of $\mathbf{H}_i$. The perturbation is due to the presence of the unmodeled tails. The algorithm initially computes matrix $\widetilde{\mathbf{\Delta D}}_i$ (using SOS of the output) that is the perturbed version of $\mathbf{\Delta D}_i$ (see section 2.1). While $\mathbf{\Delta D}_i$ is a rank-1 matrix, $\widetilde{\mathbf{\Delta D}}_i$ is not, due to the presence of the tails. $\widetilde{\mathbf{H}}_i$ is computed by applying EVD on $\widetilde{\mathbf{\Delta D}}_i$ and computing the *largest* eigenvector of $\widetilde{\mathbf{\Delta D}}_i$ (i.e. the one corresponding to the largest eigenvalue).

The perturbation analysis The goal is to find a formula that relates $\mathbf{H}_i$ to $\widetilde{\mathbf{H}}_i$. Omitting the technicalities, it suffices to mention this formula is proved to be:

$$\mathcal{E}(\mathbf{H}_i) \stackrel{\triangle}{=} \widetilde{\mathbf{H}}_i - \mathbf{H}_i = \frac{1}{\lambda_i}\,\mathbf{P}_i^{\perp}\,\mathcal{E}(\mathbf{\Delta D}_i)\,\mathbf{H}_i + \frac{1}{2\,\lambda_i^2}\left(\mathbf{H}_i^H\,\mathcal{E}(\mathbf{\Delta D}_i)\,\mathbf{H}_i\right)\mathbf{H}_i. \tag{6}$$

where λ_i is the only non-zero eigenvalue of $\mathbf{\Delta D}_i$ and $\mathbf{P}_i^{\perp}$ is the orthogonal complement of the space spanned by $\mathbf{H}_i$.

2.4 Perturbation analysis as an intermediate step in relating blind to non-blind equalizers

Although the generic framework set forth in the introduction prescribes that perturbation analysis results directly lead to conlcusions, the particular problem calls for an additional step that, nevertheless, proves to be very insightful as it relates blind to non-blind equalizers. As can be seen by inspection of (3) (non-blind case) and (5) (blind-case) any differences in the computed equalizers are due to the different RHS of the respective equations i.e. $\widehat{\mathbf{H}}_i$, $\widetilde{\mathbf{H}}_i$. In addition, the behaviour of non-blind equalizers is *already known* by the study in [4]. Thus, if blind equalizers are associated with their non-blind counterparts their behaviour in terms of equalization quality can be naturally predicted.

To this end, the *combined response*, i.e. the cascade of channel+equalizer is computed. In the blind case this is given by:

$$\widetilde{\mathbf{c}}_{L+M+1,i} = \widetilde{\mathbf{g}}_{L,i}^H\,\mathcal{T}_L(\mathbf{H}_M) = \widetilde{\mathbf{H}}_i^H\left(\mathcal{T}_L^H(\mathbf{H}_M)\right)^{\sharp}. \tag{7}$$

while in the non-blind case by:

$$\widehat{\mathbf{c}}_{L+M+1,i} = \widehat{\mathbf{g}}_{L,i}^H\,\mathcal{T}_L(\mathbf{H}_M) = \widehat{\mathbf{H}}_i^H\left(\mathcal{T}_L^H(\mathbf{H}_M)\right)^{\sharp} \tag{8}$$

where $i = 0, \ldots 2L+1$ and $\sharp$ denotes Moore-Penrose pseudoinversion. As a means to achieve the desired association the norm of the difference of the combined responses is calculated i.e. the quantity:

$$\|\widetilde{\mathbf{c}}_{L+M+1,i} - \widehat{\mathbf{c}}_{L+M+1,i}\|_2 = \left\|\left(\widetilde{\mathbf{H}}_i^H - \widehat{\mathbf{H}}_i^H\right)\left(\mathcal{T}_L^H(\mathbf{H}_M)\right)^{\sharp}\right\|_2. \tag{9}$$

Since $\widetilde{\mathbf{H}}_i = \mathbf{H}_i + \mathcal{E}(\mathbf{H}_i)$ from (6), perturbation analysis results are actively exploited in the calculations. The conclusions drawn fall into either of two classes depending on subchannel shape.

2.5 Conclusions for subchannels with no leading tails

If subchannels are such that their impulse response *begins with* "large" taps (i.e. begins with an *actual significant part* of order L^*+1) then it can be proved that:

$$\|\widetilde{\mathbf{c}}_{L+M+1,i} - \widehat{\mathbf{c}}_{L+M+1,i}\|_2 = O(\epsilon^2)\,\frac{1}{\sigma_{min}},\ i = 0, \ldots, 2L+1. \qquad (10)$$

where σ_{min} is the minimum non-zero singular value of $\mathcal{T}_L(\mathbf{H}_M)$. (Recalling the definitions in subsection 2.3, ϵ is the size of the tails). Of particular interest is the case of **effective overmodeling**. This term denotes the case where the *estimated* significant part[4] order $L+1$ is greater than the *actual* significant part order L^*+1, i.e. $L+1 > L^*+1$. In this situation, it can be proven that σ_{min} is an $O(\epsilon)$ quantity thereby causing the magnitude of the difference to be $\|\widetilde{\mathbf{c}}_{L+M+1,i} - \widehat{\mathbf{c}}_{L+M+1,i}\|_2 = O(\epsilon)$, $i = 0, \ldots, 2L+1$. In turn, this fact implies that the combined responses of blind and non-blind algorithms are *close to each other*, hence, they behave *similarly*. From the findings of [4], it is known that *good* equalization performance is expected for non-blind equalizers corresponding to delay values $i = 0, \ldots, L + (L^*+1)$ whereas for delay values $i = L+(L^*+1)+1, \ldots, 2L+1$ their behaviour is generally *poor*. As a consequence, this performance pattern is also shared by the corresponding blind equalizers.

The **exact order case**, i.e. the case where $L^*+1 = L+1$, is differentiated by the order of magnitude of $\sigma_{\min}$, which is now $O(1)$. As a result $\|\widetilde{\mathbf{c}}_{L+M+1,i} - \widehat{\mathbf{c}}_{L+M+1,i}\|_2 = O(\epsilon^2)$, $\forall i = 0, \ldots, 2L+1$. Consequently, blind and non-blind equalizers share again the same performance pattern. From the findings of [4], it is known that non-blind equalizers perform *well for every* $i = 0, \ldots, 2L+1$. Thus, blind equalizers strictly adhere to the good behaviour of their non-blind counterparts.

2.6 Conclusions for subchannels with leading tails

If subchannel impulse responses are so shaped that begin with the tails, it can be proved that:

$$\|\widetilde{\mathbf{c}}_{L+M+1,i} - \widehat{\mathbf{c}}_{L+M+1,i}\|_2 = O(\epsilon)\,\frac{1}{\sigma_{min}},\ i = 0, \ldots, 2L+1. \qquad (11)$$

where σ_{min} is the minimum non-zero singular value of $\mathcal{T}_L(\mathbf{H}_M)$. It is easily seen that (11) is differentiated by (10) of the previous section by the "$O(\cdot)$" term which now equals to $O(\epsilon)$ instead of $O(\epsilon^2)$. The order of magnitude of σ_{min} remains unaltered, being $O(\epsilon)$ in the **effective overmodeling case** and $O(1)$ in the **exact order case**.

As a consequence, $\|\widetilde{\mathbf{c}}_{L+M+1,i} - \widehat{\mathbf{c}}_{L+M+1,i}\|_2 = O(1)$, $\forall i = 0, \ldots, 2L+1$ in the **effective overmodeling case**. In other words, the compared combined

[4] Recalling the discussion in subsection 2.2, a "1-1" relationship between estimated *significant part order* and *equalizer order* is implied. In particular, a significant part of estimated order $L+1$ calls for the use of an L-the order equalizer.

responses may differ by a *lot* at *worst*. They may be closer to each other for certain delays but, in general, they diverge. Consequently, even when non-blind equalizers perform well, blind equalizers will be *poor* in performance.

In the **exact order case**, on the other hand, it holds that $\|\widetilde{\mathbf{c}}_{L+M+1,i} - \widehat{\mathbf{c}}_{L+M+1,i}\|_2 = O(\epsilon)$, meaning blind equalizers share the same performance pattern as their non-blind counterparts. Turning again to the findings of [4], it is deduced that non-blind equalizers perform well for *every* delay $i = 0, \ldots 2L+1$. As a result, blind equalizers follow perforce the same pattern and perform *well*.

2.7 Simulations

To illustrate the difference in algorithm behaviour depending on subchannel shape and reinforce what has been stated in the conclusion subsections, two simulation-based graphs are presented. They both pertain to 2-subchannel SIMO systems, each subchannel having a *total* (i.e. encompassing small and large taps alike) order of 33 . Subchannel taps are generated i.i.d. from a uniform distribution. Each subchannel has an *actual* significant part of order $L^* + 1 = 3$. The subchannels related to the experiment of fig. 1 begin with the actual significant part, whereas those associated with fig. 2 begin with 7 small taps. Another 23 small taps follow the actual significant part in this case. It is to be stressed that the *same taps* were used to synthetically generate the subchannels in both cases. Only the *ordering* of taps is different to account for different subchannel shapes. In both cases the scenario of **effective overmodeling** is examined, where the estimated significant part order is assumed to be $L + 1 = 7$. Together with combined response differences, the open eye measure (OEM) is used as a standalone measure for equalization quality. The open eye measure for a vector $\mathbf{c}$ is defined as: $\mathrm{OEM}(\mathbf{c}) = \left(\sum_i |c_i| - \max_i |c_i|\right) / \max_i |c_i|$. The lower the OEM, the better the equalization quality. In order to assist comprehension values $\epsilon/\sigma_{\min}$, $\epsilon^2/\sigma_{\min}$ are also represented as horizontal dotted lines. As can be seen by inspection of fig. 2, effective overmodeling is catastrophic when subchannels possess leading tails. In this case, no equalizers can be found that exhibit good equalization performance. On the contrary, in the absence of leading tails, good equalizers can be found for delays $i = 0, \ldots, L + (L^* + 1) = 0, \ldots, 9$, even in the effective overmodeling scenario.

3 Epilogue

This short paper outlined key elements of A. D. Beikos' PhD dissertation. The dissertation topic is the analytical assessment of the effect of channel estimation errors on telecommunication systems quality. Errors in estimating channel order and channel coefficients are considered. The analysis builds upon results of perturbation theory. The case studies are drawn from the SIMO channel equalization and the multiuser system capacity arena.

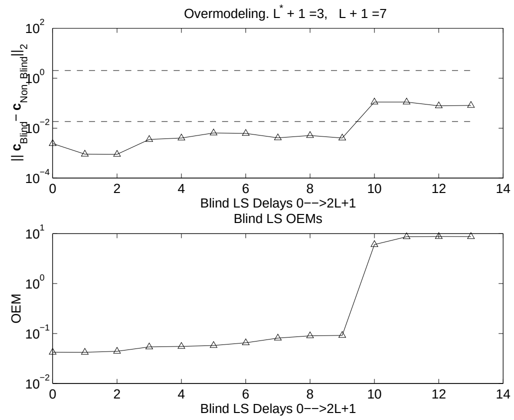

Fig. 1. Indicative results when subchannels possess *no leading tails*.

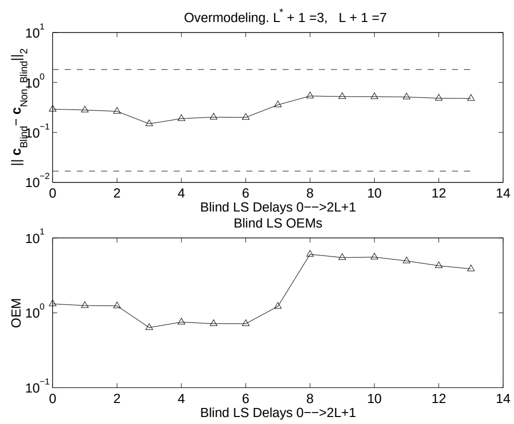

Fig. 2. Indicative results when subchannels possess *leading tails*.

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