Performance Analysis of Wireless Single Input Multiple Output Systems (SIMO) in Correlated Weibull Fading Channels

Zafeiro G. Papadimitriou *

National and Kapodistrian University of Athens
Department of Informatics and Telecommunications

zefipap@hotmail.com

November 29, 2010

Abstract

In this dissertation the statistical characteristics of the trivariate and quadrivariate Weibull fading distribution with arbitrary correlation, non-identical fading parameters and average powers are analytically studied. Novel expressions for important joint statistics are derived using the Weibull power transformation. These expressions are used to evaluate the performance of selection combining (SC) and maximal ratio combining (MRC) diversity receivers in the presence of such fading channels.

Multi-branch diversity, arbitrary correlation, Weibull fading.

I. Introduction

In recent years, the use of various telecommunication systems, their applications and usefulness to real - life has become significantly important. In

*Dissertation Advisor: Lazaros Merakos, Professor today's telecommunications era, new technologies are constantly being developed since nowadays there is a continuous need for instant access to information from a geographical point to another.

In telecommunications systems, the term "communication channel", or simply "channel", refers to either a physical transmission medium such as a wire, or to wireless connection over a physical medium such as a radio channel. As far as wireless technologies are concerned, they are being used to meet many needs such as span a distance beyond the capabilities of typical cabling, link portable or temporary workstations, overcome situations where normal cabling is difficult and/or financially impractical to remotely connect mobile users or networks. As a consequence, there is a continuing demand for increased capacity and integration of the provided services over radio channels. The radio channel though is subject to multiple phenomena such as fading that degrade the telecommunications system performance. Hence, the determination and combat of fading effects constitute an important R&D topic which is also the subject of this PhD thesis.

One of the most important effect of the fading interference is the small scale fading which results in fluctuation of the received signal's amplitude, phase and angle of arrival. In order to combat this destructive effect, in this PhD thesis diversity reception techniques are being employed. According to this method, the receiver employs more than one antennae, in order to receive multiple copies of the transmitted signal. These copies are being appropriately combined in order to satisfy network administrator demands [1].There are several diversity schemes, classified according to the combining technique employed at the receiver, the most well-known being selection combining (SC), maximal ratio combining (MRC) and equal gain combining (EGC). In diversity reception studies, it is frequently assumed that the different replicas of the same information signal are received over independent fading channels. However, in many practical wireless system applications, e.g. for small-size mobile units or indoor base stations, the receiving antennas are not sufficiently wide separated and thus the received and combined signals are correlated with each other. In order to model and analyze such realistic wireless channels with correlated fading it is mathematically convenient to use multivariate statistics [1], [2].

In the open technical literature there have been many papers published concerning multivariate distributions in relation to performance analysis of digital communication systems in the presence of correlated fading channels [3–9]. Most of these papers deal specifically with the so-called "constant" and “exponential”correlation model. For the first one, correlation depends on the distance among the combining antennas and thus this model is more suitable for equidistant antennas [1, pp. 392]. The second one, corresponds to the scenario of multichannel reception from equispaced diversity antennas. This model has been widely used for performance analysis of space diversity techniques [3] or multiple-input multiple-output (MIMO) systems. The arbitrary correlation model [4], used in our paper, is the most generic correlation model available, since it allows for arbitrary correlation values between the receiving branches. Clearly it includes the constant and exponential correlation models as special cases.

Most of the published works concerning multivariate distributions with arbitrary correlation deal with Rayleigh and Nakagami-m fading channels [2, 4–6]. In [4], new infinite series representations for the joint probability density function (PDF) and the joint cumulative distribution function (CDF) of three and four arbitrarily correlated Rayleigh random variables have been presented. In [5], expressions for multivariate Rayleigh and exponential PDFs generated from correlated Gaussian random variables have been derived, as well as a general expression in terms of determinants for the multivariate exponential characteristic function (CF). In [2] useful closed-form expressions for the joint Nakagami-m multivariate PDF and CDF with arbitrary correlation, were derived and the correlation matrix was approximated by a Green’s matrix. In a recent paper [6], infinite series representations for the PDF, CDF and CF for the trivariate and quadrivariate Nakagami-m distribution have been presented.

The Weibull distribution [10], although originally used in reliability and failure data analysis, it has been recently considered as an appropriate distribution for modeling wireless communication channels [7–9]. The main motivation of this choice is its very good fit to experimental fading channel measurements for both indoor and outdoor terrestrial radio propagation environments. In [7] it was argued that the Weibull distribution could also been considered as a generic channel model for land-mobile satellite systems. Recently, expressions for the joint PDF, CDF and the moment-generating function (MGF) for the bivariate Weibull distribution have been presented [8]. In the same reference the multivariate Weibull distribution has also been studied for the exponential and constant correlation case considering equal average fading powers. In [9] a Green’s matrix approximation for the multivariate Weibull distribution with arbitrary correlation has been presented and an analytical expression for the joint CDF has been derived. However, the performance analysis presented in [9] is restricted to SC receivers and is applicable only to the evaluation of outage probability (OP).

Motivated by the above, in this dissertation, we present a detailed and thorough analytical study of the statistical characteristics of the arbitrary correlated trivariate and quadrivariate Weibull fading distributions and their applications to various diversity receivers. For both distributions we consider the most general correlation model available, namely the arbitrary correlation model, with non-identical fading parameters or average powers and without making any approximation for the covariance matrix. In particular, novel expressions utilizing infinity series representations for the joint PDF, CDF, MGF and moments of the arbitrary trivariate and quadrivariate Weibull distributions will be presented. These analytical expressions are being conveniently used to evaluate the OP, the average bit error probability (ABEP) and other significant performance metrics for the case of SC and MRC diversity reception.

II. Results and Discussion

To investigate the trivariate and quadrivariate Weibull distributions, it is convenient to consider the multivariate Weibull distribution, $\mathbf{Z}_L = \{Z_1, Z_2, ...Z_L\}$. $\mathbf{Z}_L$ is assumed to be arbitrarily correlated according to a positive definite covariance matrix $\mathbf{\Psi}_L$, with elements $\psi_{i\kappa} = \mathbb{E}\left\langle G_i G_\kappa^* \right\rangle$, where $\mathbb{E}\left\langle \cdot \right\rangle$ denotes expectation, * complex conjugate, $i, \kappa \in \{1, 2, .., L\}$ and $\mathbf{G}_L = \{G_1, G_2, ..., G_L\}$ being joint complex zero mean Gaussian L RVs. Since $\psi_{i\kappa}$ can take arbitrary values, the analysis presented in this section refers to the most general correlation case.

A. Trivariate Weibull Distribution

For the case of the trivariate (i.e. $L = 3$) arbitrarily correlated[1] Weibull distribution and by applying the Weibull power transformation $Z = R^{2/\beta}$ [8, eq. (2)] in the infinite series representation of the Rayleigh distribution [4, eq. (5)], the novel joint PDF of $\mathbf{Z}_3 = \{Z_1, Z_2, Z_3\}$ has been derived as

[1]From now on and unless otherwise stated, it will be is assumed that the Weibull distributions under consideration are arbitrary correlated.

follows [11], [12]

$$f_{\mathbf{Z}_3}(z_1, z_2, z_3) = \frac{\beta_1\beta_2\beta_3 \det(\mathbf{\Phi}_3)}{z_1^{(2-\beta_1)/2} z_2^{(2-\beta_2)/2} z_3^{(2-\beta_3)/2}} \exp\left[-\left(z_1^{\beta_1}\phi_{11} + z_2^{\beta_2}\phi_{22} + z_3^{\beta_3}\phi_{33}\right)\right]$$
$$\times \sum_{k=0}^{\infty} \epsilon_k (-1)^k \cos(k\chi) \sum_{\ell,m,n=0}^{\infty} \frac{|\phi_{12}|^{2\ell+k}}{\ell!(\ell+k)!} \frac{|\phi_{23}|^{2m+k}}{m!(m+k)!} \frac{|\phi_{31}|^{2n+k}}{n!(n+k)!}$$
$$\times z_1^{\beta_1(\ell+n+k)+\beta_1/2} z_2^{\beta_2(\ell+m+k)+\beta_2/2} z_3^{\beta_3(m+n+k)+\beta_3/2} \tag{1}$$

where ϵ_k is the Neumann factor ($\epsilon_0 = 1, \epsilon_k = 2$ for $k = 1, 2, \cdots$), $\chi = \chi_{12} + \chi_{23} + \chi_{31}$ and $\mathbf{\Phi_3}$ is the inverse covariance matrix given by

$$\mathbf{\Phi}_3 = {\mathbf{\Psi}_3}^{-1} = \begin{bmatrix} \phi_{11}, & \phi_{12}, & \phi_{13} \\ \phi_{12}^*, & \phi_{22}, & \phi_{23} \\ \phi_{13}^*, & \phi_{23}^*, & \phi_{33} \end{bmatrix} \tag{2}$$

where $\phi_{i\kappa} = |\phi_{i\kappa}| \exp(\jmath\chi_{i\kappa})$ with $i, \kappa \in \{1, 2, 3\}$ and $|\cdot|$ denoting absolute.

By integrating (1), an infinite series representation for the CDF of $\mathbf{Z}_3$ is derived as [11], [12]

$$F_{\mathbf{Z}_3}(z_1, z_2, z_3) = \frac{\det(\mathbf{\Phi_3})}{\phi_{11}\phi_{22}\phi_{33}} \sum_{k=0}^{\infty} \epsilon_k (-1)^k \cos(k\chi) \sum_{\ell,m,n=0}^{\infty} C_3, \nu_{12}^{\ell+k/2} \nu_{23}^{m+k/2} \nu_{31}^{n+k/2}$$
$$\times \gamma\left(\delta_1, z_1^{\beta_1}\phi_{11}\right) \gamma\left(\delta_2, z_2^{\beta_2}\phi_{22}\right) \gamma\left(\delta_3, z_3^{\beta_3}\phi_{33}\right) \tag{3}$$

where $C_3 = [\ell!(\ell+k)!m!(m+k)!n!(n+k)!]^{-1}$, $\nu_{i\kappa} = |\phi_{i\kappa}|^2/\phi_{ii}\phi_{\kappa\kappa}$, $\delta_1 = \ell + n + k + 1$, $\delta_2 = m + \ell + k + 1$, and $\delta_3 = n + m + k + 1$ with $\gamma(\cdot, \cdot)$ denoting the incomplete lower Gamma function [13, eq. (3.381/1)].

The joint MGF of $\mathbf{Z}_3$ can expressed as $M_{\mathbf{Z}_3}(s_1, s_2, s_3) = \mathbb{E}\langle \exp(-s_1 Z_1 - s_2 Z_2 - s_3 Z_3)\rangle$. From (1) and following the integral solutions using the Meijer G-function presented in [8, pp. 3610], the following novel expression has been obtained [11], [14]

$$
\begin{aligned}
M_{\mathbf{Z}_3}(s_1, s_2, s_3) &= \beta_1\beta_2\beta_3 \det(\mathbf{\Phi}_3) \sum_{k=0}^{\infty} \epsilon_k (-1)^k \cos(k\chi) \\
&\times \sum_{\ell,m,n=0}^{\infty} C_3 \frac{|\phi_{12}|^{2\ell+k} |\phi_{23}|^{2m+k} |\phi_{31}|^{2n+k}}{s_1^{\beta_1(\ell+n+k+1)} s_2^{\beta_2(\ell+m+k+1)} s_3^{\beta_3(m+n+k+1)}} \\
&\times \Upsilon\left[\frac{\phi_{11}}{s_1^{\beta_1}}, \beta_1(\ell+n+k+1)\right] \Upsilon\left[\frac{\phi_{22}}{s_2^{\beta_2}}, \beta_2(\ell+m+k+1)\right] \\
&\times \Upsilon\left[\frac{\phi_{33}}{s_3^{\beta_3}}, \beta_3(m+n+k+1)\right]
\end{aligned}
\tag{4}
$$

where $\Upsilon(\cdot)$ is given in [8, eq. 8].

B. Quadrivariate Weibull Distribution

For the case of the quadrivariate (i.e. $L = 4$) Weibull distribution, we consider the inverse covariance matrix, $\mathbf{\Phi}_4$, expressed as

$$
\mathbf{\Phi}_4 = \mathbf{\Psi}_4^{-1} = \begin{bmatrix} \phi_{11}, & \phi_{12}, & \phi_{13}, & 0 \\ \phi_{12}^*, & \phi_{22}, & \phi_{23}, & \phi_{24} \\ \phi_{13}^*, & \phi_{23}^*, & \phi_{33}, & \phi_{34} \\ 0, & \phi_{24}^*, & \phi_{34}^*, & \phi_{44} \end{bmatrix}
\tag{5}
$$

where the $\phi_{i\kappa}$ $i, \kappa \in \{1, 2, 3, 4\}$ can take arbitrary values with the restriction of $\phi_{14} = \phi_{14}^* = 0$. Although this restriction is a mathematical assumption, necessary for the derivation of the equivalent statistics and does not correspond to a physical explanation, it is underlined that our approach is more general than of [15] for the multivariate Rayleigh distribution. More specifically, the statistical properties derived in [15] hold only under the assumption that $\mathbf{\Psi}$ is tridiagonal, i.e. when $\phi_{i\kappa} = 0$ for $|i - \kappa| > 1$. The same assumption was used in [9], where the correlation matrix was approached by the tridiagonal Green matrix.

In principle, an expression for the joint PDF of $\mathbf{Z}_4 = \{Z_1, Z_2, Z_3, Z_4\}$ can be derived using [4, eq. (16)] and by applying the power transformation described in [8, eq. (2)] as a product of the modified Bessel function of the first kind $I_n(u)$. However, this approach will not be adopted since expressions containing modified Bessel functions are difficult to be mathematically manipulated, e.g. performing integrations. Instead, a more convenient approach is to use its infinite series expansion [13, eq. (8.447/1)]. Thus, the following PDF has been obtained [11]

$$f_{\mathbf{Z}_4}(z_1, z_2, z_3, z_4) = \beta_1\beta_2\beta_3\beta_4 \det(\mathbf{\Phi}_4) \exp\left[-\left(z_1^{\beta_1}\phi_{11} + z_2^{\beta_2}\phi_{22} + z_3^{\beta_3}\phi_{33} + z_4^{\beta_4}\phi_{44}\right)\right]$$
$$\times \sum_{j=0}^{\infty} \sum_{k=-\infty}^{\infty} \epsilon_j (-1)^{j+k} \cos(A) \sum_{\ell,m,n,p,q=0}^{\infty} C_4 \left|\phi_{12}\right|^{2\ell+j} \left|\phi_{13}\right|^{2m+j} \left|\phi_{24}\right|^{2n+|k|} \left|\phi_{34}\right|^{2p+|k|} \left|\phi_{23}\right|^{2q+|j+k|}$$
$$\times z_1^{\beta_1(\ell+n+j+1)-1} z_2^{\beta_2[J_1]-1} z_3^{\beta_3[J_2]-1} z_4^{\beta_4(n+p+|k|/2+1)-1} \tag{6}$$

where $C_4 = [\ell!(\ell+j)!m!(m+j)!n!(n+|k|)!p!(p+|k|)!q!(q+|k+j|)!]^{-1}$, $A = j(\chi_{12}+\chi_{23}+\chi_{31})+k(\chi_{23}+\chi_{34}+\chi_{42})$, $J_1 = \ell+n+q+(j+|k|+|j+k|)/2+1$ and $J_2 = m+p+q+(j+|k|+|j+k|)/2+1$.

Following a similar procedure as before and by using (6), the corresponding CDF and MGF have also been obtained [11], [16].

C. Performance Analysis

In this section important performance criteria for diversity receivers with three or four arbitrarily correlated diversity branches operating over Weibull fading and additive white Gaussian noise (AWGN) channels will be presented. In particular, by using the previously derived expressions for the statistical characteristics of the trivariate and quadrivariate Weibull distribution, the performance of MRC and SC diversity receivers have been studied and their OP and ABEP have been derived.

For the system model considered, the equivalent baseband signal received at the ℓth branch can be mathematically expressed as $\zeta_\ell = wh_\ell + n_\ell$ where w is the complex transmitted symbol having average energy $E_s = \mathbb{E}\langle|w|^2\rangle$, h_ℓ is the complex channel fading envelope with its magnitude $Z_\ell = |h_\ell|$ being a Weibull distributed RV and n_ℓ is the AWGN with single-sided power spectral density N_0. The instantaneous, per symbol SNR, of the ℓth diversity channel is $\gamma_\ell = Z_\ell^2 E_s/N_0$, while its average is $\overline{\gamma}_\ell = \mathbb{E}\langle Z_\ell^2\rangle E_s/N_0 = \Gamma(d_{2,\ell})\Omega_\ell^{2/\beta_\ell} E_s/N_0$ where $d_{\tau,\ell} = 1+\tau/\beta_\ell$ with $\tau > 0$. Note that it is straightforward to obtain expressions for the statistics of γ_ℓ by replacing at the previously mentioned expressions for the fading envelope Z_ℓ, β_ℓ with $\beta_\ell/2$ and Ω_ℓ with $(\alpha_\ell\overline{\gamma}_\ell)^{\beta_\ell/2}$ [8]. Thus, denoting $\boldsymbol{\gamma_L}$=$\{\gamma_1, \gamma_2, ...\gamma_L\}$, and since the CDF $F_{\boldsymbol{\gamma_L}}(\gamma_1, \gamma_2, ..., \gamma_L)$ and the MGF $M_{\boldsymbol{\gamma_L}}(s_1, s_2, ..., s_L)$ of the SNR for the trivariate and quadrivariate

Weibull distribution can be easily obtained, but will not be presented here due to space limitation.

1) Performance of MRC Receivers

For MRC receivers the output, per symbol, SNR ($\mathrm{SNR_o}$), is $\gamma_{mrc} = \sum_{\ell=1}^{L} \gamma_\ell$ [1]. To obtain the ABEP performance it is convenient to use the MGF-based approach. Hence, the MGF of the L-branch MRC output can be derived as $M_{\gamma_{mrc}}(s) = M_{\boldsymbol{\gamma_L}}(s, s, .., s)$. By using the MGF-based approach, the ABEP of noncoherent binary frequency-shift keying (NBFSK) and binary differential phase-shift keying (BDPSK) modulation signaling can be directly calculated. For other types of modulation formats, numerical integration is needed in order to evaluate single integrals with finite limits.

2) Outage Probability of SC Receivers

The instantaneous SNR at the output of a L-branch SC receiver, will be the SNR with the highest instantaneous value between all branches, i.e. $\gamma_{sc} = \max\{\gamma_1, \gamma_2, ..., \gamma_L\}$ [17]. Since the CDF of γ_{sc}, $F_{\boldsymbol{\gamma_{sc}}}(\gamma_{sc}) = F_{\boldsymbol{\gamma}}(\gamma_{sc}, \gamma_{sc}, ..., \gamma_{sc})$, P_{out} can be easily obtained as $P_{out}(\gamma_{th}) = F_{\gamma_{sc}}(\gamma_{th})$ for both trivariate and quadrivariate cases.

D. Performance Evaluation Results

Using the previous mathematical analysis, in this section performance evaluation results for the SC and MRC receivers will be presented. Non-identical distributed Weibull channels, i.e., $\overline{\gamma}_\ell = \overline{\gamma}_1 \exp[-(\ell-1)\delta]$ where δ is the power decay factor are considered and for the convenience of the presentation, but without any loss of generality, $\beta_\ell = \beta \;\forall\; \ell$ will be assumed. Considering a triple-branch diversity receiver with the linearly arbitrary normalized covariance matrix[2] given in [2, pp. 886] and SC diversity, the OP has been obtained as a function of the first branch normalized outage threshold $\gamma_{th}/\overline{\gamma}_1$ for different values of β and δ. The performance evaluation results, illustrated in Fig. 1, indicate that P_{out} degrades with increasing $\gamma_{th}/\overline{\gamma}_1$ and δ and/or decreasing β. Note that for $\beta = 2$ and $\delta = 0$ the obtained results are in agreement with previously known performance evaluation results presented in [9].

[2]Note that the covariance matrix specifies the fading correlation between two complex Gaussian RVs.

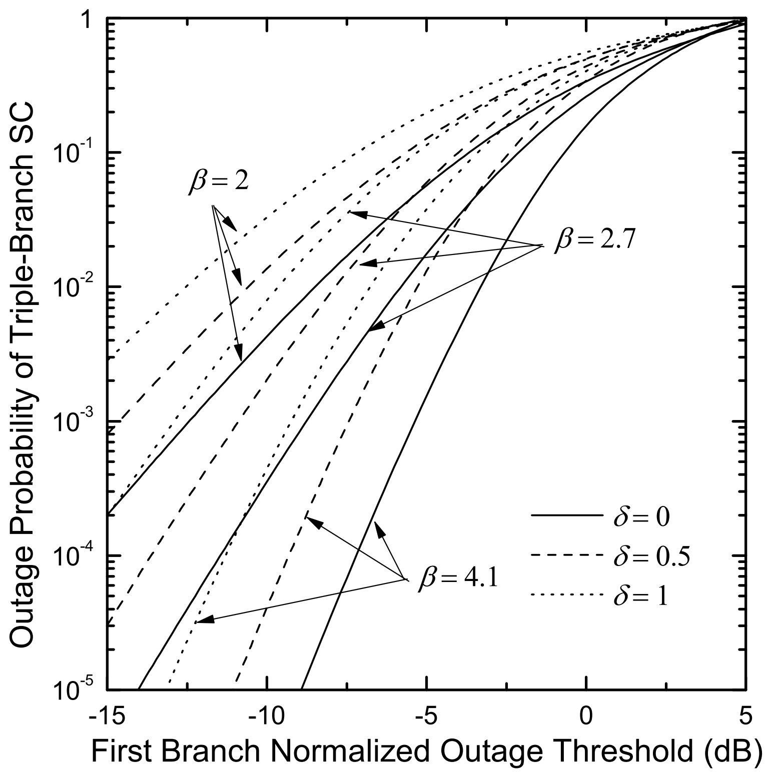

Figure 1: Outage probability of triple-branch SC receiver as a function of the first branch normalized outage threshold for different values of β and δ.

For MRC receiver and BDPSK signaling, the ABEP has been obtained and is illustrated in Fig. 2 for four receiving branches, assuming the covariance matrices presented in [4, eq. (34)]. As expected, the ABEP improves as the first branch average input SNR $\overline{\gamma}_1$ increases, while for a fixed value of $\overline{\gamma}_1$, similar to the SC diversity, a decrease of β and/or an increase of δ degrades the ABEP. Furthermore, performance evaluation results obtained by means of computer simulation also shown in Fig. 2 and have verified the accuracy of the analysis. It is finally noted that for the four-branch diversity reception and $\overline{\gamma}_1 > 5$ dB, only one term is required to achieve accuracy better than 10^{-5}.

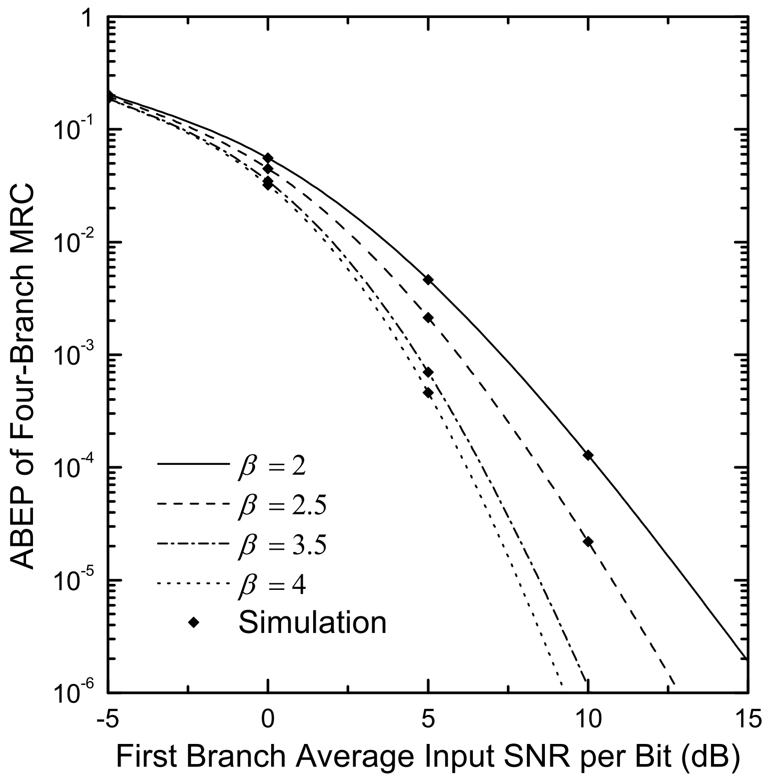

Figure 2: ABEP of four-branch MRC receiver as a function of the first branch average input SNR per bit for different values of β.

III. Conclusions

In this dissertation the novel statistical characteristics of the trivariate and quadrivariate Weibull fading distribution with arbitrary correlation, non-identical fading parameters and average powers have been derived using infinite series representations. The theoretical analysis has been also applied in order to evaluate the performance of SC and MRC diversity receivers.

References

[1] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, 2nd ed. New York: Wiley, 2005.

[2] G. K. Karagiannidis, D. A. Zogas, and S. A. Kotsopoulos, "An efficient approach to multivariate Nakagami-m distribution using Green's matrix approximation," *IEEE Trans. Wireless Commun.*, vol. 2, no. 5, pp. 883–889, Sep. 2003.

[3] Y.-K. Ko, M.-S. Alouini, and M. K. Simon, "Outage probability of diversity systems over generalized fading channels," *IEEE Trans. Commun.*, vol. 48, no. 11, pp. 1783–1787, Nov. 2000.

[4] Y. Chen and C. Tellambura, "Infinite series representation of the trivariate and quadrivariate Rayleigh distribution and their applications," *IEEE Trans. Commun.*, vol. 53, no. 12, pp. 2092–2101, Dec. 2005.

[5] R. K. Mallik, "On the multivariate Rayleigh and exponential distributions," *IEEE Trans. Inf. Theory*, vol. 49, no. 6, pp. 1499–1515, Jun. 2003.

[6] P. Dharmawanse, N. Rajatheva, and C. Tellambura, "Infinite series representation of the trivariate and quadrivariate Nakagami-m distributions," *IEEE Trans. Wireless Commun.*, vol. 6, no. 12, pp. 4320–4328, Dec. 2007.

[7] P. S. Bithas, G. K. Karagiannidis, N. C. Sagias, P. T. Mathiopoulos, S. A. Kotsopoulos, and G. E. Corazza, "Performance analysis of a class of GSC receivers over nonidentical Weibull fading channels," *IEEE Trans. Veh. Technol.*, vol. 54, no. 6, pp. 1963–1970, Nov. 2005.

[8] N. C. Sagias and G. K. Karagiannidis, "Gaussian class multivariate Weibull distributions: Theory and applications in fading channels," *IEEE Trans. Inf. Theory*, vol. 51, no. 10, pp. 3608–3619, Oct. 2005.

[9] G. C. Alexandropoulos, N. C. Sagias, and K. Berberidis, "On the multivariate Weibull fading model with arbitrary correlation matrix," *IEEE Antennas Wireless Propag. Lett.*, vol. 6, pp. 93–95, 2007.

[10] W. Weibull, "A statistical distribution function of wide applicability," *Appl. Mech. J.*, no. 27, 1951.

[11] Z. G. Papadimitriou, P. T. Mathiopoulos, and N. C. Sagias, "The trivariate and quadrivariate weibull fading distributions with arbitrary correlation and their applications to diversity reception," *IEEE Trans. Commun.*, vol. 57, no. 11, pp. 3230–3234, Nov. 2009.

[12] Z. G. Papadimitriou, N. C. Sagias, P. S. Bithas, P. T. Mathiopoulos, and L. Merakos, “The trivariate weibull distribution with arbitrary correlation,” in *IEEE International Workshop on Satellite and Space Communications*, Leganes, Spain, Dec. 2006, pp. 249–253.

[13] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 6th ed. New York: Academic Press, 2000.

[14] Z. G. Papadimitriou, P. S. Bithas, P. T. Mathiopoulos, N. C. Sagias, and L. Merakos, “Triple-branch mrc diversity in weibull fading channels,” in *International Workshop on Signal Design and Its Applications in Communications*, Chengdu, China, Sep. 2007, pp. 247–251.

[15] L. E. Blumenson and K. S. Miller, “Properties of generalized Rayleigh distributions,” *Ann. Math. Statist.*, vol. 34, pp. 903–910, 1963.

[16] Z. G. Papadimitriou, P. T. Mathiopoulos, N. C. Sagias, and L. Merakos, “On the weibull distribution with arbitrary correlation,” in *3rd International Symposium on Communications, Control and Signal Processing*, St. Julians, Malta, Mar. 2009.

[17] G. K. Karagiannidis, D. A. Zogas, and S. A. Kotsopoulos, “Performance analysis of triple selection diversity over exponentially correlated Nakagami-m fading channels,” *IEEE Trans. Commun.*, vol. 51, no. 8, pp. 1245–1248, Aug. 2003.