Game-theoretic analysis of networks.

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Abstract. Algorithmic mechanism design is an important area between computer science and economics. One of the most fundamental problems in this area is the problem of scheduling unrelated machines to minimize the makespan. The machines behave like selfish players: they have to get paid in order to process the tasks, and would lie about their processing times if they could increase their utility in this way. The problem was proposed and studied in the seminal paper of Nisan and Ronen, where it was shown that the approximation ratio of mechanisms is between 2 and n.

In this thesis, we present some recent improvements of the lower bound to $1+\sqrt{2}$ for three or more machines and to $1+\varphi$ for many machines. Since the gap between the lower bound of 2.618 and the upper bound of n is huge, we also propose an alternative approach to the problem, which first attempts to characterize all truthful mechanisms and then study their approximation ratio. Towards this goal, we show that the class of truthful mechanisms for two players (regardless of approximation ratio) is very limited: tasks can be partitioned in groups allocated by affine minimizers (a natural generalization of the well-known VCG mechanism) and groups allocated by threshold mechanisms.

Finally we generalize a tool we have used in the proof of the $1+\sqrt{2}$ lower bound: we give a geometrical characterization of truthfulness for the case of three tasks, which we believe that might be useful for proving improved lower bounds and which provides a more complete understanding of truthfulness.

1 Introduction

A social choice is a single joint decision which is made after taking into account the preferences of different individuals affected by the decision. The most common examples of social choice are voting, auctions and government policy. Mechanism design takes into account the selfish strategic behavior of the single individuals (in a game theoretic sense) in order to design an algorithm or protocol that makes this social choice. The reason we need mechanism design is that the preferences of the individuals are private.

Algorithmic mechanism design is an important area between computer science and economics. The two most fundamental problems in this area are the problem of scheduling unrelated machines [30] and the problem of combinatorial auctions [22, 14, 7]. Here we are dealing with the scheduling problem, but our main result which is the characterization of truthful mechanisms for two players extends naturally to the more general domain of combinatorial auctions.

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In the scheduling problem, there are n players (machines) and m tasks to be executed on these machines. Each task j needs time t_{ij} on machine i. We want to allocate the tasks to machines in a way that minimizes the makespan (the time required to finish all tasks). The problem is that the machines are selfish and will not reveal the true values (we assume that only machine i knows the true values t_{ij}).

When we depart from the classical design of algorithms and try to extend it to mechanisms, we face the problem that these algorithms have to deal with selfish agents, who may not be truthful. This restricts the repertoire of available algorithms and brings forth the question of what kind of mechanisms are available in this framework.

A mechanism consists of two parts, the allocation algorithm and the payment functions, one for each player. Each player i declares its own execution times t_i. The mechanism collects all the declarations t and allocates the tasks according to an allocation function $a : R^{n\times m} \to \{1, \dots, n\}^m$ from the set of all execution times to the set of partitions of m tasks to n players. It is more convenient to denote an allocation using the characteristic variables: a_{ij} is an indicator variable for task j to be allocated to machine i. The mechanism also pays each player i a payment p_i. The payment depends on the declared values t and indirectly on the allocation. A mechanism is truthful, if no player has incentive to lie. We are dealing here with the standard and more restricted notion of truthfulness, dominant truthfulness, in which a player has no incentive to lie for every value of the other players. It is well-known that in truthful mechanisms, the payment to player i depends on the values t_{-i} of the other players and on the allocation a_i of player i: $p_i = p_i(a_i, t_{-i})$.

2 Dissertation Summary

In the first chapter we give an introduction to Mechanism design. From the second chapter and until the end of the Thesis we deal specifically with the scheduling selfish unrelated machines problem as all results in this thesis concern this specific problem. In the second chapter we state the problem and start presenting some relevant fundamental facts and results. We consider a number of different approaches to it. On the way we also present some of the tools we have developed for studying the problem [12, 19]. Then we present all known algorithms for the problem and show some easy lower bounds namely a 2-lower bound for the case of two machines and an n lower bound for some special classes of algorithms. Two of these classes are the class of additive and the class of threshold mechanisms. We show that the class of threshold mechanisms is identical to the class of additive mechanisms (we announced the result in [11] and here we provide the complete proof).

In the third chapter we explore the possible truthful mechanisms for the scheduling problem. We develop some tools for the general case of m tasks but we only manage to characterize the possible mechanisms for the case of three tasks. We believe that this work provides a better intuition about truthful mechanisms.

As the existing lower bound for the case of 3 players uses the result for the two-task case, this work is potentially useful not only for characterizing truthful mechanisms for more than two players, but also for obtaining new lower bounds. This chapter is based on an unpublished working paper.

In the fourth chapter we give a proof a lower bound $1 + \sqrt{2} \approx 2.41$ on the approximation ratio of truthful mechanisms for the case of three or more machines. This chapter is based primarily on the paper "A lower bound for scheduling mechanisms" [12] co-authored with George Christodoulou and Elias Koutsoupias. (The same result can also be found in [8, 10] however the proof presented here is different and shorter.)

In the fifth chapter we give a proof of a $1 + \phi \approx 2.618$ lower bound as the number of machines n tends to ∞. Our technique also gives improved lower bounds for any constant number of machines $n \geq 4$. This chapter is based primarily on the paper "A lower bound of $1 + \phi$ for truthful scheduling" [19] co-authored with Elias Koutsoupias.

The objective in the scheduling problem is to minimize the makespan, i.e. to minimize the maximum completion time, or in other words to minimize the L_∞ norm of the machine loads. On the other hand the goal achieved by the VCG, which is the best known algorithm, is that of minimizing the sum of completion times, in other words to minimize the L_1 norm of the machine loads. It turns out that our techniques can be easily adapted in order to get lower bounds for all L_p norms $2 \leq p < \infty$.

Finally in the last chapter we provide a characterization for the case of two-players and arbitrarily many tasks. We show that the class of truthful mechanisms is very limited: A decisive truthful mechanism partitions the tasks into groups so that the tasks in each group are allocated independently of the other groups. Tasks in a group of size at least two are allocated by an affine minimizer and tasks in singleton groups by a threshold mechanism (which is however a task-independent mechanism except for countably many points). This characterization is about all truthful mechanisms, including those with unbounded approximation ratio. A direct consequence of this approach is that the approximation ratio of mechanisms for two players, for the objective of minimizing the makepan, is 2, even for two tasks. In fact, it follows that for two players, VCG is the unique algorithm with optimal approximation 2. This characterization provides some support that any decisive truthful mechanism (for 3 or more players) partitions the tasks into groups some of which are allocated by affine minimizers, while the rest are allocated by a threshold mechanism (in which a task is allocated to a player when it is below a threshold value which depends only on the values of the other players). This chapter is based primarily on the paper "A characterization of 2-player mechanisms for scheduling" [11] co-authored with with George Christodoulou and Elias Koutsoupias.

3 Related work

The scheduling problem on unrelated machines is one of the most fundamental scheduling problems [17, 18]. The problem is NP-complete. Lenstra, Shmoys, and Tardos [24] showed that it can be approximated in polynomial within a factor of 2 but no better than 3/2, unless P=NP.

Nisan and Ronen introduced the mechanism-design version of the problem in the paper that founded the algorithmic theory of Mechanism Design [30, 31]. They showed that the well-known VCG mechanism, which is a polynomial-time algorithm and truthful, has approximation ratio n. They conjectured that there is no deterministic mechanism with approximation ratio less than n. They also showed that no mechanism (polynomial-time or not) can achieve approximation ratio better than 2. We improved it to $1 + \sqrt{2}$, in [10, 12] and further to $1 + \varphi$ in [19].

Nisan and Ronen [30] also gave a randomized truthful mechanism for two players, that achieves an approximation ratio of 7/4. Mu'alem and Schapira [28] proved a lower bound of $2 - \frac{1}{n}$ for any randomized truthful mechanism for n machines and generalized the mechanism in [30] to give a $7n/8$ upper bound. Recently Lu and Yu [25] gave a 1.67-approximation universally truthful randomized algorithm improving it later on [26] to a 1.59-approximation algorithm.

In another direction, [9] showed that no fractional truthful mechanism can achieve an approximation ratio better than $2 - 1/n$. It also showed that fractional algorithms that treat each task independently cannot do better than $(n + 1)/2$ and this bound is tight.

In very recent paper [5] Ashlagi, Dobizinski and Lavi prove a lower bound of n for a special class of mechanisms, which they call "anonymous".

Lavi and Swamy [23] considered the special case of the same problem when the processing times have only two possible values low or high, and devised a deterministic 2-approximation truthful mechanism. Very recently Yu [33] generalized their results constructing a randomized $7(1 + \epsilon)$-approximation algorithm for the case when the processing times belong to $[L_j, L_j(1 + \epsilon)] \cup [H_j, H_j(1 + \epsilon)]$ where $L_j < H_j$ and $\epsilon < 1/16mn$.

Another special case of the problem is the problem on related machines in which there is a single value (instead of a vector) for every machine, its speed. Myerson [29] gave a characterization of truthful algorithms for this kind of problems (one-parameter problems), in terms of a monotonicity condition. Archer and Tardos [4] found a similar characterization and using it obtained a variant of the optimal algorithm which is truthful (albeit exponential-time). They also gave a polynomial-time randomized 3-approximation mechanism, which was later improved to a 2-approximation, in [2], and very recently to a PTAS by Dhangwatnotai, Dobzinski, Dughmi and Roughgarden [13]. These mechanisms are truthful in expectation. Auletta De Prisco, Penna and Persiano [6] provided a deterministic, monotone $(4 + \epsilon)$ approximation algorithm for the case of constant number of machines m. Andelman, Azar, and Sorani [1] improved this to a FPTAS and additionally gave a 5-approximation algorithm for arbitrary m. Kovács improved the approximation ratio to 3 [20] and to 2.8 [21].

Saks and Yu [32] proved that, for mechanism design problems with convex domains of finitely many outcomes, which includes the scheduling problem, a simple necessary monotonicity property of the allocations of different inputs (and without any reference to payments) is also sufficient for truthful mechanisms, generalizing results of [16, 22]. Monderer [27] showed that this result cannot be essentially extended to a larger class of domains. Both these results concern domains of finitely many outcomes. There are however cases, like the fractional version of the scheduling problem, when the set of all possible allocations is infinite. For these, Archer and Kleinberg [3] provided a necessary and sufficient condition for truthfulness which generalizes the results of [32].

4 Results and Discussion

Perhaps the most important result of this thesis is the characterization of truthful mechanisms for two players. Seeking a characterization is unrefutably a very natural and important question to ask but can we not prove a lower bound for the scheduling problem without getting our hands dirty with a characterization?

There are basically two directions one can follow for providing a lower bound for this problem:

The first approach is to use, an appropriately selected, small subset of the input instances. Fix one instance and consider all its possible allocations (providing a finite approximation ratio). Then argue how each one of the possible allocations results to approximation ratio at least r for some other instance from our chosen set. This is possible because the Monotonicity Property gives a condition that should be satisfied by any two instances of the problem and their corresponding allocations, hence allows us to show how the allocation of one instance affects the allocation of other instances. This approach had been already followed in [30, 10] using a finite set of small instances of 2 and 3 machines respectively and no more than 5 tasks.

Another (more ambitious) approach is to provide a global characterization of all possible mechanisms, considering all possible inputs, which are infinitely many. After this it is very easy to determine the mechanism with the best approximation ratio. This approach however solves a potentially more difficult problem. The only characterization we know until now, came very recently. For the case of two machines [15] Dobzinski and Sundararajan show that every finite approximation mechanism is task-independent, while in the next section we provide a characterization of all (regardless of approximation ratio) decisive truthful mechanisms in terms of affine minimizers and threshold mechanisms. Until now the only example of a new lower bound obtained by a characterization is the lower bound of 2 for instances with two (or more) tasks [11]. (It is however considerably easier to prove the same lower bound for instances with 3 or more tasks [30] without employing a characterization.)

The proof of the $1 + \varphi$ lower bound does not use a characterization, but in some sense lies somewhere in-between these two different approaches in the sense that it uses an infinite subset of the input and a sophisticated double induction to keep track of how all these allocations depend from each other.

The discouraging thing is that, despite using infinitely more players and tasks, the improvement on the lower bound achieved is very small. This might be considered as an indication that however difficult the characterization approach might be, it is our only serious hope for proving the Conjecture by Nisan and Ronen [30].

5 Conclusions

Our work has given the first steps towards the resolution of a central problem in algorithmic mechanism design about ten years after the problem was posed. The gap between the lower bound and the upper bound is however still huge. We believe that the characterization approach we propose is the right one for resolving this very important problem.

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