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## ABSTRACTS OF DOCTORAL DISSERTATIONS



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### **The Committee of Research and Development**

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## PREFACE

This volume includes extended abstracts of Doctoral Dissertations conducted in the Department of Informatics and Telecommunications, University of Athens, and completed from 1/2015 to 12/2015.

We publish this volume to demonstrate the breadth and quality of the original research performed by our Ph.D. students and faculty and to facilitate the dissemination of their innovative research results. We are happy to present the 11<sup>th</sup> yearly collection of this kind and expect this initiative to continue in the years to come. The submission of an extended abstract in English is required by all graduating doctoral students in our Department.

We would like to thank all graduates who contributed to this volume and hope that this was a positive experience for them. Finally, we would like to thank PhD candidate Nikos Bogdos for his help and attention to detail in putting together this volume.

The painting in the cover is called “*Venetian Lantern*” contributed by artist *Katerina Vasilaki*.

The DiT Dept. Committee on Research and Development Activities

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Athens, May 2016



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# Models and methodologies for forecasting and evolution of diffusion and competition of telecommunications markets

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**Abstract.** This thesis presents a methodological framework which integrates in a uniform way several important parameters influencing the progress of penetration of telecommunications products and services. Given the above methodological framework, significant problems of high-end market are faced, regarding diffusion and competition. The approaches developed are based on an appropriate mathematical and statistical background and they are applied to specific case studies, providing highly accurate results.

This thesis involves the valuation and prediction of the demand for telecommunications products and services, as well as the competition on the telecommunications market, with the use of novel methodologies based on an appropriate mathematical and statistical background. The basic scope is the improvement of the prediction capability compared to existing wide used prediction models of the specific scientific area. The total contribution involves the creation of new methodologies and the application of existing ones in the new relevant fields of application.

The objective of this thesis corresponds to an important part of the techno-economic design aiming to the prediction of demand and competition in the telecommunications market. Examples of such markets are the developing broadband fiber networks and wireless mobile terminals. The design of these networks together with their expected use in the future, are important elements related to the development of the corresponding infrastructure. Due to the rapidly developing technologies and the growing demand for access, design of these networks should provide and support innovative network services and technologies. Determining the number of users, the expected utilization of services, the volume of mobile data and the shaped market shares due to competition, should be the drivers for the development of the infrastructure to support the network operation.

Both the methodologies and their results obtain added value for the case of Greece, as the country is going through a period of rapid developments in terms of telecommunications infrastructure and services development.

\* Dissertation Advisor: Thomas Sphicopoulos, Professor

# **1 Dissertation Summary**

## **1.1 The problem that the thesis investigates and its primary targets – the significance of the results**

The objective of this thesis corresponds to an important part of the general techno-economic design that accompanies the development and function of the telecommunications networks. It involves the review and forecasting of the diffusion of telecommunications services and products, as well as of the competition in telecommunications markets.

This thesis presents a rigorous methodological framework that integrates a uniform manner several important parameters that influence the course of penetration of telecommunications products and services. Based on the above methodological framework useful results emerged as to predict the demand for services and products, and competition in the high-tech market. These results emerged as the methodologies applied to specific case studies.

The results of this thesis, besides the high scientific interest that they present, have extra practical value under the prism of the modern markets, which are characterized by the rapid development of the broadband optical networks and of the mobile telecommunications.

The design of modern communications networks and the prediction of future penetration of the services to be provided through these networks are critical elements related to the course of development of the infrastructure. Due to rapidly developing technologies and the ever growing demand for access requirements, the design of these networks should provide and support innovative network services and technologies. The determination of the number of users, intensive use of services, the volume of mobile data and market share due to competition should be a guide for the development of infrastructure that will support the operation of the networks. In order to become the design, development and deployment of network infrastructure should necessarily precede the precise identification and prediction of intention to use the services to be provided. Otherwise there is a strong probability that a failure of planning and investment, the new network will not receive the expected acceptance and use.

The study addresses this very necessary stage of development of communication infrastructures. Particularly in the case of Greece, the results acquire added value as the country goes through a period of rapid developments in the modernization of telecommunications infrastructure and services. In addition, the boundaries between voice networks and data networks have ceased to exist, marking the convergence of electronic communications networks. The development of the Internet is a prime example of this, as with a conventional computer the average user is allowed access to services such communication (using video and audio) and data management (TV, video, radio, etc.) through the same network infrastructure. It is obvious that the convergence of networks and the emergence of new services leads to increased resource requirements and infrastructure, and to increased number



of users. Consequently, there is need for improvement and development of appropriate methodologies to address the new problems that have arisen.

Due to developments and changes that were performed on the telecommunications market, as described previously, relating both to market liberalization and the rapid technological developments, it becomes clear the need for elaboration of demand forecast studies and their integration in the respective business plans a modern telecommunications provider, in order for him to enter the market, to ensure its sustainability and to maintain or increase its share. The forecast of demand plays an important role in the administration and management of an organization, and can provide information on the technological changes that may affect their corporate objectives, identify the medium- and long-term activities and to highlight the importance disposal a technological innovation, the pace of technological substitution generations and other critical issues.

Moreover, the effect of increased competition in the telecommunications market and the continuous emergence of new products led to increasing problems associated with forecasting the demand facing both the providers and regulators and other stakeholders. The providers and manufacturers failed to produce reliable forecasts for demand for products or services, on which to base their business plans, can have dramatic effects that can be associated with either failure or by an excess supply on the market.

Rapid technological progress has resulted in the provision of a multitude of new products and services, while the market has not been accompanied by corresponding research for the study of penetration in the relevant market. The relevant research work has remained largely in the form of monopoly of the telecommunications market, in which only the active state, and dominant provider in which the incidence of technological innovations was quite slower. Consequently, it is now necessary to research activity in this direction in order to address current problems relating to demand forecasting.

The forecast of demand remains a critical factor because it is part of a broader economic analysis of the telecommunications market becoming a key input in the coming stages. The most representative case is usually treated in the telecommunications market which is directly dependent on the results of prediction of penetration is the dimensioning of the network and other infrastructure should be developed to support the growing demand for the services offered. The above example shows that the accuracy of the results of forecasting is an extremely critical factor, mainly because of its link with the required investments.

The major contemporary problems relating to the provision of the telecommunications market demand and that highlight the need for developing further research in this area are described below.

The first of these problems concerns the description of the very modern telecommunications market, which now is oligopolistic or competitive, which no longer includes only the sovereign, government, providers and alternative providers. Current services are not only limited to fixed voice communications, but also include mobile communications and data communications, and video.

In addition, the boundaries between voice networks and data networks have ceased to exist, marking the convergence of electronic communications networks. The Internet is the most typical example of this, as with a conventional computer the average user is allowed access to services such as communication (using video and audio) and data management (TV, video, radio, etc.) through the same network infrastructure.

For the incumbent, who until now operated in a monopolistic market, increasing handling of voice and data was disturbed by the market entry of alternative providers. This leads to research questions, in order to consider the new form of market that arose. Considering however that the telecommunications market is characterized by specific entry barriers (market entry barrier), there is a need to develop new approaches and adapting existing ones in order to study the relevant competition issues arising.

As a direct consequence, there is higher competition between providers on the pricing policy in order to maintain and increase their market share. By extension, the disposal value of a service is expected to have a direct influence on the penetration, even creating a field of research which concerns the study of price elasticities and advertising on the service demand.

The emergence and rapid spread of mobile technology has created further research issues related to the substitutability and complementarity of the services provided. For example, there are still uniquely identified questions regarding whether the mobile telephony gradually replaces the stable.

Another important category of contemporary problems in the field of forecasting demand in the telecommunications market relates to the time range of forecasts. In this connection there are two directions of research interest: short-term and long-term prediction, which face different demand. In the direction of short-term forecasting, among the problems is the prediction of the movement of the users of providers (churn effect) and burden of network traffic, aiming primarily to determine the economic impact and the use of an appropriate range of infrastructure and staff to meet demand needs. The long-term demand forecasting includes issues relating mainly to the potential entry of new providers in the market, the progress of penetration of new products and services and levels of market saturation in demand service. The prediction of the course of penetration of a new product on the market is an extremely important and complex problem, given the rapid development of technology, which has resulted in the continuous emergence of new or differentiated products and services, which often are not available recorded history data to predict the subsequent course.

All these issues are both research and practical problems involving all stakeholders in the telecommunications market: the regulators, providers, manufacturers, and even end users. Furthermore, raise research interests, around which the existing literature is incomplete, thus substantiating the need for further research in the same area.

## **1.2 Other relevant research efforts that have been nominated in the bibliography**

There is a large number of studies focusing on modeling the diffusion of innovations, aiming to provide accurate estimates and forecasts. The increasing academic interest in this area began in the 1960s, when a significant number of related papers

came to light. Fourt and Woodlock [1], Mansfield [2], Floyd [3], Rogers [4], Chow [5] and Bass [6] were the first to consider the modeling of a technology's diffusion. Their work has encouraged many modifications often adopted and studied, even in recent research efforts.

Time series were mainly studied under a deterministic prism, until Yule [7] introduced the notion of stochasticity in 1927. According to him, every time-series approach can be regarded as the realization of a stochastic process. This simple idea launched a number of time-series methods, varying in parameter estimation, identification, model checking and forecasting. Nevertheless, it was the work of Box and Jenkins in their publication *Time Series Analysis: Forecasting and Control* [8] that integrated the existing knowledge and made a breakthrough in the area. The Box-Jenkins approach is a coherent, versatile, three stage iterative cycle for time series identification, estimation and diagnostic checking. The evolution of computers made the use of autoregressive integrated moving average (ARIMA) models popular and applicable in many scientific fields.

The gap in research concerning the comparative performance of sales forecasting models in a given situation was underlined by Armstrong, Brodie and McIntyre [9]. Furthermore, the use of ARIMA models has not been widely investigated in the case of forecasting the diffusion of innovations. In addition, Meade [10] stated in 1984 that the popular diffusion models are among the heavily used forecasting techniques in a corporate environment. The forecasting accuracy of ARIMA models has been compared with a selection of diffusion models, among which the models of Floyd, Bass and Gompertz [11,12], by Gottardy and Scarso [13].

Exponential smoothing methods have been around since the 1950s, and are still among the most popular forecasting methods used in business and industry. Nevertheless, exponential smoothing can be used with any discrete set of repeated measurements. The major advantage of these models in providing mainly accurate short term forecasting has surprisingly led to lack of research regarding their application in long-term diffusion forecasting. Gardner and McKenzie made a breakthrough in the research area of exponential smoothing with a series of papers ([14–16]) by developing new versions of the Holt–Winters methods that damp the trend as the forecast horizon increases. Their work has stimulated the interest in this area, resulting to the inclusion of the damped trend exponential smoothing methods in the successfully applied approaches in empirical studies, as discussed by Gardner [17]. In addition, the damped trend is recommended by Armstrong [17] as a well established forecasting method that should improve accuracy in practical applications. The area of exponential smoothing has undergone a substantial revolution in the recently past years.

During the analysis of an ecosystem, the equilibrium analysis of dynamic predator–prey systems is one of the most important stages, which is described based on appropriate systems of Lotka–Volterra equations [19], the latter used to model the system dynamics [20]. The Lotka–Volterra equation, which was developed to model the interaction between the two competing species based on the logistic curve, is considered as an alternative competitive diffusion model for analysing the telecommunications market. Applications of the Lotka–Volterra equations to the analysis of technology diffusion in a competitive market can be found in relevant literature [21–23].

### **1.3 The main original results that risen from the thesis**

The main original results that have risen from the thesis are:

- Development of a comprehensive framework for assessing and forecasting the demand in the telecommunications market, with emphasis on innovative use of time series models, independently or in combination with traditional aggregate demand models, covering all phases of diffusion. This framework is both applied and markets high technology products in general.
- Valuation methods for selecting appropriate, on a case-demand models.
- Overview of social, economic and other parameters (such as pricing) that affect the process of diffusion of a product or service in the telecommunications market with case studies.
- Implementation of chronological Grey model with a minimum of chronological data (four) at the beginning of the diffusion of innovation and verification of its effectiveness against a cumulative classic diffusion model [24].
- Creation of a diffusion model for the initial phase of diffusion of innovation that incorporates predictor of pricing based on the corresponding service of the past, that proves superior to the simple form of the cumulative diffusion model which was based.
- Combination of forecasting cumulative diffusion models and time series ARIMA models to achieve better prediction of the classical cumulative diffusion models in the middle phase of diffusion [25].
- Creation of a variant chronological cumulative trend model Holt's incorporating a cumulative forecasting parameter affected by a diffusion model in the final phase of diffusion in order to achieve improved provision against cumulative diffusion models [26].
- Development of models to assess and predict the level of competition and the concentration of the telecommunications market, based on procedures from the natural world and in particular the population biology (population biology), in which market competition is simulated with survival course of species in an environment [27-28].
- Creation of a new predictive model market share of a new entrant wireless provider using tools such as The HHI competition and diffusion models [29].
- Investigation of the use of time series models for short-term forecasting of various telecommunications markets competition indicators.

## **2 Results and discussion**

The thesis is structured in 11 chapters. The first chapters present the concepts, methodologies, assumptions and approaches that constitute the common background of the following chapters of the thesis.

After the first introductory chapter, which gives a brief description of the investigation carried out, the second chapter follows which describes developments in telecommunications technology and in social, economic and regulatory factors affecting them, as well as a review of the history and development of telecommunications,

including with the reasons which made forecasting the demand and competition as a necessary part in the preparation of business plans in telecommunications.

The third chapter takes provides a brief review and classification of existing methodologies in the literature for the estimation and prediction of the demand. It contains a detailed description of the approach adopted for the preparation of this thesis, which is related to the diffusion theory and aggregate models, i.e. the mathematical models that describe it. These are then used in the thesis for creating an appropriate methodological framework which is implemented and evaluated the study characteristics specific cases.

The fourth chapter is extensive reference to the time series, the use of which is a key piece of research conducted in the framework of this thesis. It describes basics of time series and their use to export forecasts, as the stochastic process, measures and time series analysis, stagnation and simple time series models. Then an extensive report on time series models used in the following chapters of the thesis is presented in the context of foresight studies. First a description is provided of stationary stochastic processes with a focus on integrated autoregressive moving average models ARIMA and time series analysis with their use. Then the smoothing methods are analyzed, namely the methods of the simple and the double moving average, simple and double exponential smoothing, exponential smoothing by adjusting the voltage and exponential smoothing by adjusting the voltage and seasonality. Finally reference is made to the existing provisioning software.

The fifth chapter initially presents the application of Grey theory to produce short-term forecasts of the diffusion of a new high technology in the early stages of the process. The valuation methodology is performed using real diffusion data from European countries. After obtaining the minimum data points for the application of Grey theory (four data), a classic diffusion model, the Gompertz model, and the GM (1,1) model, are applied to the sample, leading to results that verify the accuracy of the forecasts after applying the Grey theory. The second study in the initial stage of diffusion presents a new methodology that offers short-term predictions of the diffusion of a new technology in the early stages of the process. Once enough data is gathered to make a time-series, it is applied to sample both models. A classic cumulative diffusion model and a variant of using the phenomenon of price reduction based on estimates of an older similar technology are used. This approach is based on the assumption that the influence of the price cut should be provided for the initial diffusion, even if such data are not yet available. This reduction phenomenon should be based on experience gained from previous similar cases of telecommunications technology.

The sixth chapter presents an innovative methodology combining time series and cumulative diffusion models to predict short diffusion process in the middle phase of the procedure where the data available are limited. Projections incorporate the influences of the ARIMA model and the cumulative diffusion models and are providing more accurate than any standalone application of each approach. This process avoids the use of simple means. It also avoids the use of weights (weights) as a method of combining, as it would include personal assumptions, while correlations between forecast errors are likely to change from period to period. Two popular telecommunications innovations were chosen for the implementation of the methodology (broad-

band and mobile telecommunications). The new methodology provided better predictions from each model separately in both technology and in each geographical application. Forecast combinations is a topic widely researched in the field of statistics. The one-year-ahead forecasts of each approach were compared based on three widely used measures of accuracy: the Mean Square Error (MSE), the Mean Absolute Error (MAE) and the Mean Absolute Percentage Error (MAPE), with the MAPE being the main measure, as noted in other similar studies of forecast combinations. Even though the new methodology provided better predictions than each model separately in both technologies and for every application, all three approaches provide more-or-less reliable predictions for the period considered. Another important observation is that the forecasting accuracy of the ARIMA model diminishes gradually at this stage of the growth process, from period to period, whereas the corresponding predictions of the Linear Logistic model improve. The differences in numbers may not seem of great importance. Nevertheless, these small differences represent, in reality, some thousands of new subscribers. Even though the forecasting power of the methodology seems limited, it should be taken into consideration that the forecasting improvement is for one-year horizon. This single year's improved forecast could make the difference in the sense of corporate competition, as this knowledge is a useful guideline for the upcoming year's strategy programming. The use of ARIMA models and their combination with the cumulative diffusion models not been widely explored in forecasting the diffusion of innovations, so this study collect the largest number of citations of the studies resulting from this thesis so far.

The seventh chapter presents a new methodology that offers improved long-term forecasts of the diffusion of an innovation compared with two classical cumulative diffusion models, the model of Gompertz and the linear logistic model. After collecting sufficient number of historical data for analysis by the exponential smoothing method and since diffusion has reached the inflection point of the sigmoid curve, the method of Holt's damped trend is applied, directed by the forecasted planned saturation point of a cumulative diffusion model. The application of the method to broadband diffusion data on all OECD countries and the United States for different forecasting horizons confirmed the accuracy of prediction. Application of the method in the case of broadband penetration in the total sum of the OECD countries and the United States from 1997 until 2009, for different forecasting horizons ranging from 6 months up to 30 months forth, verified its accuracy and illustrated its performance capabilities. The forecasts of each approach were compared based on the three widely used measures of accuracy estimation, comparison and forecasting. After its validation with the hold back sample, the proposed method was applied for a 48-month forecasting horizon, until mid 2013. The proposed approach method predictability extends to long-term forecasts. This variant combines the advantages of both approaches, the cumulative diffusion models affected by the expected sigmoidal form of the process, while the exponential smoothing methods rely mainly on newly recorded historical data. This is the first time that this combination is applied for creating an innovative variant of the model of Holt, inspired and designed for the specific diffusion region.

Chapter eight describes the methodology for assessing and forecasting the development of competition in the telecommunications market and generally of high tech markets, based on the principles of population biology, which deals with the study of the evolution of the species population in the context of interaction that develops between populations and the influence of the environment in which they are located. It is this evolution of the species is in concordance with the course providers in an oligopolistic or competitive market, such as telecommunications, within the meaning of the respective market shares. In this chapter concepts of competing species (predator - prey) and a description of the respective model are presented.

The ninth chapter deals with the valuation methodology of Lotka - Volterra for the evolution of the concentration of the telecommunications market in two innovative fields of application, that of fixed telephony (PSTN) and mobile prepaid - contracts. The mathematical description of the methodology was performed using the Lotka-Volterra model, in a prey-predator mode and the corresponding parameters were estimated by the means of genetic algorithms. The proposed methodology showed itself capable of estimating market equilibrium and market concentration and it can be applied over any high technology market meeting the above characteristics, providing valuable inputs for managerial decisions, strategic planning and regulatory decisions to the players of a high technology market. The results output includes the ability of the methodology to accurately assess the extent of competition in the telecommunications market and its concentration and predict the balance point in the future. The evaluation methodology was performed using genetic algorithms, while the evaluation of the results was based on classical statistical error measures.

In the tenth chapter, special topics of the telecommunications market competition are developed. More specifically, in the first part, competition concentration indicators of the telecommunications market are calculated and the use of time series for forecasting is investigated, with an application to the providers of the Greek mobile telephony. More specifically, the commonly used mobile communications market concentration indicators are described, analyzed and evaluated for the case of the Greek mobile market, which is considered a typical representative of oligopolistic market. Moreover, forecasting using time series is applied in order to test the accuracy of short-term forecasting of such measures, which are directly affected by market events, such as mergers. The second part introduced an empirical model that predicts the later entrant's market share development in the mobile telecommunications market of a country. It can be used as a preliminary tool by the analysts of the mobile market, as well as by the potential investors interested in predicting the market's potential before the act of entry, merger or acquisition. The evolution of market share is explained by factors such as the moment of entry, the HHI at the moment of entry and the forecasted changes in the penetration rate, as estimated by the Linear Logistic model. There is strong evidence that the earlier a new operator enters the country's market, the greater potential the operator has in gaining significant market share. The model gives an overall idea of the market share development of a later entrant that can become more accurate if combined with other methods, which take into account other crucial variables, such as pricing, cost and profits. The most logical extension of the model is the inclusion of the pricing effect. The difficulty lays in the absence of an-

nounced price time-series data. The effect of the marketing potential of each operator would also improve the forecasting accuracy of the model, although the recording of such a value demands too much market information. Nevertheless, the predicted outcome of the model's application approximates the actual data, especially the «long – run» market share of the two operators.

Finally, in the final chapter, the general conclusions of the thesis as well as future research directions are presented.

### **3 Conclusion**

The results that came up from the development and evaluation of the above mentioned methodologies provide important and direct contribution to the development of business plans and to the determination of critical decisions. They offer the capability of precise assessment and forecasting of the diffusion of an innovation, as well as of the competition level of telecommunications markets, taking into consideration the most critical factors that affect the markets' dynamics.

Regarding the researching extensions of this thesis, they evolve around the creation of a common, coherent methodological framework that deals with the phenomenon of the diffusion of telecommunications' innovations, taking into consideration all the parameters that affect it. In order to achieve the production of accurate results, it is essential to unite all the influences that are placed. The success of a diffusion process of a new product or service in a market depends directly on a great variety of factors. This methodological framework will include all the parameters that were examined in this thesis, as well as the stochastic analysis of the results. The implementation of the above mentioned framework should be accompanied by the development of the proper software so that the execution of the necessary mathematical calculations would be automatic.

In reference to the methodologies that deal with study of the competition in the telecommunications' markets and are based on the principles of population biology, among the possible research extensions is the assessment of their capabilities in other high-technology markets, that have similar characteristics with the telecommunications' market, like oligopolistic form and barriers for entrance of new players. An important extension of the Lotka-Volterra methodology is the addition of extra stochastic terms in the initial mathematical equations.

Finally, an important extension that can be applied almost to every methodology that has been presented is the inclusion of decision parameters in them, such as price and advertising, in order to evaluate their influence in the diffusion process of a telecommunications product, together with the rest of the parameters that are included in the definitions of the models.

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# Decision Making Methods and Uncertainty Modeling for the Development of a Roadmap for Future Home Networks

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**Abstract.** In recent years there is a need for supporting broadband services in home and office networks. Towards this end, there has been great progress in technologies of high bandwidth, which resulted to an interest for the evolution of home networks and many standards have been developed for the interconnection of the devices. Given the increasing technological development, it would be interesting to investigate the technical solutions and the possibility to achieve high bit rate connectivity. Towards this end, this thesis aims to investigate the development of a roadmap as a key for ensuring the smooth development of future home networks, and also prescribe their course and determine the most appropriate technological solutions for the development of such systems. However, apart from technical difficulties, the network designer should take into consideration the different economic and social issues affecting the adoption of home network systems. In view of such difficulties, the need for a roadmap of home network systems arises in order to address all these issues.

This thesis investigates the major aspects affecting the development of future home networks and creates an effective roadmap for their development based on multicriteria decision making methods. Although most decision makers rely on decisions based partly on intuition, such an approach is not effective especially in a demanding business environment. It is therefore particularly important to investigate the effectiveness of these methods under different conditions, taking into account the influence of uncertainty that may undermine these processes.

This thesis aims both to contribute to a roadmap implementation in next-generation home networks, based on multi-criteria decision making and also study critical issues of uncertainty lurking in decision making processes and may affect the final result. The study and the development of the above lies on the Analytical Hierarchy Process (AHP), and especially in pairwise comparison method.

## 1 Dissertation Summary

Home Networking Systems (HNS) will play a crucial role in achieving end-to-end broadband service provision, enabling the penetration of the future internet. Traditionally, in-building networks, for instance in corporate or academic settings, have a tenfold higher capacity than their access points to the rest of the telecommunication infrastructure. Since Fiber-To-The-Home (FTTH) promises

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symmetric access data rates of at least 100 Mbit/s per household [1], HNS should support gigabit per second data transmission, limiting the latency time below 10msec. Inside new buildings, optical fiber systems may provide the ultimate solution in terms of bandwidth and range. However, installing fiber cables inside older buildings is usually not preferred, since it is accompanied with increased cost and user discomfort.

### 1.1 Roadmap for Future Home Networks

Considering the vision of future HNS, there is a variety of technological solutions that could contribute to the development and further growth of next-generation HNS, to achieve high speed connectivity. Such solutions include Ethernet, Power Line Communications (PLC), Wifi, 60 GHz and the optical solutions wireless or wired.

In the context of home networking, extension is a fundamental functionality. As shown in Figure 1, network extension aims to extend the HNS coverage. Conceivably, every network device may extend the network acting as a “multi-hopper”. The extender device may act as a router by forwarding data for other network nodes and can also generate and receive packets of supporting ad-hoc network functionalities.

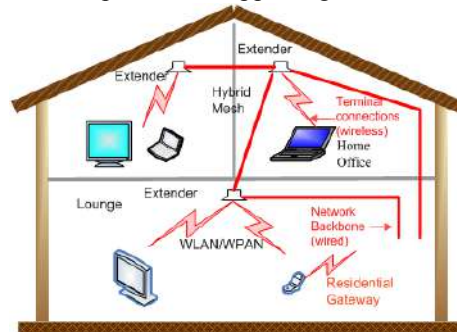


Figure 1. Illustration of hybridization of technologies inside the HN

There are various technology solutions that could be considered for network extension, each with their own particular characteristics. Radio systems, such as WiFi, are already commercially available and 802.11n promises up to 600Mb/s data rates using multiple-input multiple output (MIMO) techniques [2], but the actual network throughput should be lower. Systems operating at the unlicensed 60GHz band [3] and short range Ultra Wide Band (UWB) [4] systems can provide higher data rates. Wireless gigabit-per-second transmission is feasible at 60GHz, but such systems have not reached full technological maturity. State-of-the-art PLC [5] provide hundreds of megabit-per-second wireline connectivity using the already installed power cables of the house. Extending them in the gigabit regime is however a challenge because of the particularities of the power line channel. Optical Wireless Systems (OWS) [1] may also provide high data rates, possibly reaching 10Gb/s in the future, in either the infrared or visible spectrum region. However, there are still important technical limitations. In Line-of-Sight (LOS) OWS, blocking is a serious issue. In diffuse configurations (no need for LOS path), one obtains a poorer signal to noise ratio.

The above discussion illustrates that future HNS will probably consist of hybrid systems and technology roadmapping is key in ensuring their smooth deployment. This thesis seeks to shed light in this problem, contributing to the technological roadmapping of HNS in general. At first, we evaluate various crucial technological and socio-economic issues that affect the deployment of future HNS taking into account popular broadband services such as HDTV and VoIP. Moreover, focusing on the extension functionality, we consider three possible alternative technologies for network extension namely IEEE 802.11n, 60GHz radio and PLC. To rank these alternatives, the AHP methodology is used [6] as a fundamental part of an effective

technology roadmapping introduced in [7]. The main objective is to evaluate the prospects of the various home networking technologies both from a technical and a socio-economic point-of-view.

## 1.2 Optical House Vision

Given the large capacity of optical communications systems in long haul and metropolitan area networks, one could think of using optics at the home network as well [8]. Short range, fiber systems [9] may be used inside the home to ensure that no bottlenecks will occur in end-to-end HN services. However, one should also bear in mind that optical technologies are faced with a different set of requirements when deployed inside the customer premises. For example, in existing buildings, residents would be eager to avoid new cable installations which may come at an increased cost and discomfort.

OW systems [10] are already being widely applied in point-to-point outdoor connections in the access network [11]. Recent years have seen a growing interest in indoor applications as well, in both the infrared (IR) [12] and the visible portion [13] of the spectrum. The latter technology, also known as visible light communications (VLC), relies on white light emitting diodes (LEDs) which are used to provide illumination and communication simultaneously. The advantages of OW technology include the virtually unlimited available bandwidth, its inherent security (since the electromagnetic field at these frequencies cannot pass through walls) and limited interference with domestic appliances and conventional radio communication systems. In addition, provided that certain eye safety regulations are met, electromagnetic radiation at these wavelengths is safe, since mankind has been exposed to it for centuries, because of the Sun. Multi-Gb transceivers have already been developed and commercialized for fiber systems. However, since the issue of cost is of paramount importance near the customer premises, low-cost components should be used.

The above considerations illustrate that OW may constitute a good candidate for enabling the delivery of many broadband services such as HDTV and Web 2.0 applications including content sharing, on-line gaming, etc. Unlike conventional radio wireless systems however, OW is not a mature technology and there are several issues concerning their deployment that remain unclear. For example the choice of backbone HN supporting Gb/s. PLC can offer an interesting solution. Alternatively, if one wants to abandon the no-new-wire concept, optical fibers can be used for hotspot interconnection. Another issue to resolve is what the actual system to be used for wireless connections. Do we rely on IR or VLC or both?

The above issues are complicated by the fact that technology penetration depends on a blend of economic, social and performance-related criteria [14]. According to the above, in this thesis we also attempt to shed some light in this complex problem, using AHP. Five alternative deployment scenarios are identified and ranked based on the findings of several carefully designed pairwise comparisons. The importance of the various criteria involved in the deployment of the network are evaluated and discussed. The obtained results form a key part of the optical wireless roadmap.

## 1.3 Uncertainty Issues in Decision Making

A fundamental problem in decision making is to grade the importance of a set of alternatives and assign a weight to each of them. The importance of alternatives usually depends on several criteria which can be evaluated within the decision making framework in which pairwise comparisons (PWC) are an essential ingredient [15]. PWC enables the ranking of alternatives by allowing the experts to compare the various criteria or alternatives in pairs instead of assigning their priorities in a single

step [16]. This reduces the influence of subjective point of views, associated with eliciting weights directly. The influence of uncertainty due to the imperfect and subjective expert judgments is of paramount importance when considering the credibility of the outcome of a decision making process. Several studies have attempted to shed some light on this issue in the context of PWC. [17]-[21].

The main purpose of our work is to provide a suitable characterization of the impact of uncertainty in PWC. A first step in order to characterize the impact of uncertainty in PWCs, is to identify a suitable measure for quantifying its effects. Assume for instance that  $N$  different alternatives are pairwise compared by  $M$  experts, each with possibly a different view on the ranking of the alternatives. PWC aims at providing an average ranking, encompassing all these diverse opinions of the experts. It is of course natural to expect that the credibility of the overall process will be increased as the size of the expert group increases. Therefore, one possible way of measuring the trustworthiness of the results is to define the probability of rank reversal ( $P_{RR}$ ) as follows: Let  $W_1, \dots, W_N$  be the weights calculated by the PWC in the case of a very large group of experts ( $M \rightarrow \infty$ ). In a practical situation where  $M$  is finite, uncertainty may undermine the PWC and the calculated weights  $w_k$  may turn out different than  $W_k$ . Uncertainty can be due to the difference of opinion among the experts or inconsistent pairwise comparisons. The probability of rank reversal is defined as:  $P_{RR} = P\{\text{the ranking obtained by } w_i, 1 \leq i \leq N, \text{ is different than that of } W_i\}$ .

According to the aforementioned, we propose an uncertainty model to address two major issues concerning the probability of rank reversal in pairwise comparisons. The first issue highlights the relations between the probability of rank reversal and the group size of experts participating in surveys. The procedure is based in Monte Carlo simulations (MC).

Although this is a valid approach, it is often preferable to have a theoretical model for estimating the  $P_{RR}$ . A theoretical model is often much more straightforward to implement and requires much less computational time than Monte Carlo simulations. It can also form a solid basis for understanding and extending the PWC application framework. So as an extension to the aforementioned numerical model of estimating the  $P_{RR}$ , this thesis also discusses how the probability of rank reversal can be estimated theoretically. We show that instead of using MC,  $P_{RR}$  is estimated through the multivariate normal cumulative distribution function (MVNCDF).

## 2 Results and Discussion

### 2.1 Towards a Roadmap for future Home Networks

This thesis explains various critical technological and socio-economic issues in order to investigate their influence on the development of future home networks. In this section, the first steps towards a roadmap for the next generation home network have been undertaken. Based on pairwise comparison surveys conducted within the ICT-OMEGA project consortium [22], a number of technical, economic and social issues determining the penetration of future home networks have been evaluated and prioritized. It was shown that experts rate social acceptance of primary importance for successful product commercialization. Within this criterion, health issues have proven the main concern, possibly reflecting some public skepticism on biological effects of electromagnetic radiation. It seems that as time goes by, health issues will crucially affect the deployment strategies for home networks. Regarding performance issues, coverage was deemed as the most important performance measured followed by downstream bit rate. If fiber installation is not an option, the existing technologies (radio, PLC, OW or some hybrid alternative) may provide a broadband alternative

each with its own merits and drawbacks however. Compatibility with existing solutions and home appliances was also highly weighted. From the economic point-of-view, the installation cost turned out to be the major factor. Wireless solutions and even PLC are compatible with the “no new wire” approach and could therefore a hybrid solution can lead to reduced installation costs. Various requirements for HDTV and VoIP, envisioned to be major service application scenarios for future home networks have been considered and weighted.

Furthermore, we analyze the creation processes for a roadmap, associated with home networking systems, and specifically to the functionality of the network extension. It was shown that future HNS will probably consist of hybrid systems and technology roadmapping is key in ensuring their smooth deployment. This section aims to contribute towards this end, by ranking three possible alternative technologies for network extension, namely IEEE 802.11n, Radio 60GHz and PLC. OW systems are not considered in this work, since the majority of the experts believed that this technology is still relatively quiet immature. For example, it remains unclear which OW configuration will prevail from a number of possible choices, including VLC and IR systems which can be LOS, diffuse or both. Such issues prevent reliable forecasting of several performance-related measures. Furthermore UWB was not considered in the HNS roadmap since they can provide high bandwidth short range links at small distances, but at distances longer than 10m the throughput is comparable to that of 802.11g.

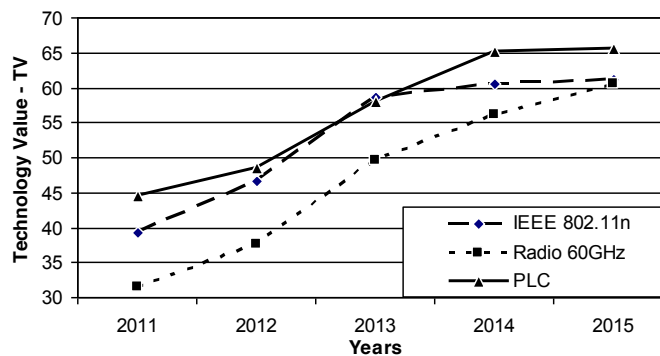


Figure 2. TV for the technology alternatives for network extension

The results, presented in Figure 2, focus on network extension but are also indicative for the rest of the home network components as well. AHP indicates that PLC takes some precedence over the wireless alternatives. 802.11 is ranked second best while 60GHz system are regarded as a longer term alternative, which could provide gigabit per second connectivity. In this thesis the merits of hybrid integration of these technologies, either in the PHY or a higher network layer are also discussed. Indeed, PLC would be the most ubiquitous connection in the home, while all the devices would include additional interface to support the benefits of intelligent switching to include a wireless extension. Hybrid radio/PLC systems, combining the merits of both technologies could also provide an alternative. Products combining both technologies at the PHY have already been commercialized. The issue of hybridization is more interesting when the PLC and the 60GHz systems are combined together: 60GHz can provide high bandwidth wireless connection within the room while PLC can be used to extend the connectivity across multiple rooms of the house. In order for PLC to provide the future home network backbone however, its capacity must be extended in the Gbps regime. A sensitivity analysis was also performed in order to estimate the uncertainties involved and it was deduced that they do not in general undermine the results of the ranking.

## 2.2 Envisioning Optical House Scenario

This thesis also focuses on the evaluation of the potential OW technologies in the development of home network, towards the vision of a future home or office network based entirely on optical systems.

Towards this end, we consider five different scenarios. The first one scenario ( $A_1$ ) relies on bidirectional IR line-of-sight connections and a PLC backbone. The second scenario ( $A_2$ ) is similar to  $A_1$  with the exception that VLC lamps are also providing with downstream connectivity, enhancing the coverage of the overall system. The VLC and IR subsystems can be combined at a higher network layer such as the MAC layer thereby constituting a hybrid system. Note also that in this scenario, VLC is used for downstream data only. The other two architectures  $A_3$  and  $A_4$  are similar to  $A_1$  and  $A_2$  respectively except that the plastic optical fiber (POF) is used at the backbone instead of PLC. Architectures  $A_1$  and  $A_2$  have the merit of remaining compatible with the no-new-wires approach. Both PLC and POF are expected to achieve Gb/s connections in the near future. All the aforementioned wireless alternatives are not yet commercially available solutions but have been successfully demonstrated in the lab. The fifth alternative ( $A_5$ ) is to extend the POF connections right up to the user terminals. No wireless connections are provisioned in this scenario.

In this section, an evaluation of the potential of OW technologies for home/office network deployments was carried out. A number of important findings were obtained which must form part of any type of carefully designed roadmap for optical home networking technologies. The first finding concerned the identification and ranking of various factors and criteria determining the deployment of such systems. It was made clear, that since home networking systems will be placed at the customer premises, there are many social and economic factors that must be taken into account. Social aspects were shown to be of paramount importance and health issues can provide a serious incentive for installing IR and VLC hotspots which are inherently safe. The results obtained by the surveys were justified taking into account the nature of the optical wireless systems. Next, we identified and ranked the five architecture scenarios using the AHP framework. The results suggest that a combination of VLC and IR hotspots, along with a PLC backbone provides the most favorable option, but is closely followed by IR hotspots connected with a PLC backbone. In any case, the results clearly indicated the advantages of PLC backbone in terms of ease of installation in older buildings. The ranking results were also further elaborated using sensitivity analysis and MC simulation. It was found that under uncertainty the priorities of the alternatives are correlated and this correlation reduces the  $P_{RR}$ . The hybrid VLC/IR with PLC backbone solution is almost never surpassed by the IR PLC alternative, even if all parameters are randomly perturbed by  $\pm 10\%$ .

This section provided a framework to identify factors that could speed up or impede the deployment of optical communication technologies in the home network. It is our hope that it would constitute a first step for bridging the gap between the important research work carried out in the field, and the socio economic requirements that will guarantee the business prospects of their wide scale deployment.

## 2.3 Convergence Properties and Practical Estimation of $P_{RR}$

The present section chapter attempts to deal with both points, presented in 1.3. We first develop a model for incorporating uncertainty in PWC and consider a suitable measure for quantifying the uncertainty level. We then discuss how  $P_{RR}$  varies with the group size  $M$  depending on the uncertainty level and extract several interesting conclusions from this variation. It is shown that there is not much sense in using more than  $M=15$  experts in the decision making process because the rate of decrease of  $P_{RR}$  is already small for  $M > 15$ . We then address the issue of how  $P_{RR}$  can be estimated



from just the values of the pairwise comparison matrices  $\mathbf{P}^{(m)}$  obtained by the experts. Given this information, we discuss a numerical method for estimating  $P_{RR}$  based on Monte Carlo simulation. The results indicate that for a sufficiently large group of experts, one can obtain a reasonable approximation to the actual value of  $P_{RR}$ .

Concerning the first issue of how this probability is reduced by augmenting the number of experts participating in the surveys, the results dictate that there is not much to be gained by increasing the number of experts beyond 15, even if uncertainty level is large, as shown in the example presented in Figure 3. This seems to hold regardless the number of criteria, the level of uncertainty, the weight estimation method and other parameters changed in our model.

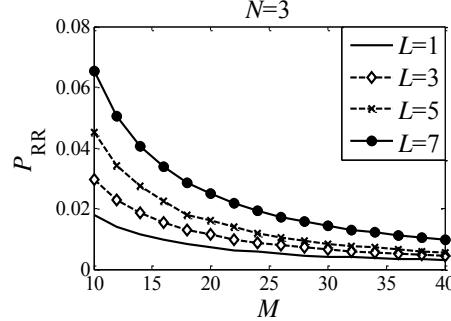


Figure 3.  $P_{RR}$  as a function of the number of experts for different levels of uncertainty  $L$

Using perturbation theory we have argued that the convergence of the probability of rank reversal with the number of experts is not crucially dependent on the uncertainty statistics. We have also shown numerically that the choice of comparison scale and weight selection method does not significantly affect the convergence. Regarding the second issue of how the  $P_{RR}$  can be estimated in practice, from the elements of the PWC of a single expert group, two alternative methods have been discussed for extracting information on the statistical behavior of the uncertainty-induced perturbations. The first one is based on the average value, while the second one on the standard deviation of the actual data. It was shown that standard deviation provides reasonable good accuracy and can therefore be used in practical applications of the method for estimating the credibility of the outcome. Interestingly enough, this conclusion holds under other situations such as fuzzy pairwise judgments, alternative preference scales, weight estimation methods and accounting for a different uncertainty level for each expert in the group.

#### 2.4 Theoretical Estimation of $P_{RR}$

In this thesis a theoretical model is also proposed, based on MVNCDF, in order to estimate the probability of rank reversal and investigate the impact of uncertainty in the final outcomes of pairwise comparisons. Towards this end, we have introduced uncertainty in the opinion of experts developing the corresponding model, taking into account  $s_w$  and  $s$  as the perturbation strengths of the perturbations of each expert from the ideal weights and PWC matrices, respectively. This approach is formulated for two alternative weight estimation methods: the eigenvalue (EV) and the geometric mean (GM) method. We show that the MVNCDF yields accurate results compared to MC simulations regardless of the number of criteria and the weight estimation method used.

We consider  $N$  alternatives and their ideal weights  $W_k$  are chosen so that  $W_{k-1} \geq W_k$  for  $2 \leq k \leq N$ . In the event of no rank reversal, one would therefore have  $w_{k-1} \geq w_k$ . In the case of the EV, the average perturbations of the weights are determined as  $\delta\omega_k = w_k - W_k$

and in the absence of any rank reversal we will have  $W_1 + \delta w_1 \geq W_2 + \delta w_2 \geq \dots \geq W_N + \delta w_N$ . The probability of no rank reversal  $P_n$  is therefore  $P_n = P(\delta w_1 - \delta w_2 > W_2 - W_1, \dots, \delta w_{N-1} - \delta w_N > W_N - W_{N-1})$  while the probability of rank reversal is simply  $P_{RR} = 1 - P_n$ . Since  $y_k = \delta \omega_k - \delta \omega_{k-1}$  will follow a Gaussian-like distribution,  $P_{RR}$  can be approximated by the MVNCDF, once the covariance matrix and the mean values of  $y_k$  are determined. It is easy to see that  $\langle y_k \rangle = \langle \delta \omega_k \rangle - \langle \delta \omega_{k-1} \rangle$  and the covariance matrix is

$$C_{\kappa\mu} = \langle y_\kappa y_\mu \rangle - \langle y_\kappa \rangle \langle y_\mu \rangle = c_{\kappa\mu} - c_{\kappa, \mu-1} - c_{\kappa-1, \mu} + c_{\kappa-1, \mu-1} - \langle y_\kappa \rangle \langle y_\mu \rangle \quad (1)$$

where  $c_{ij} = \langle \delta \omega_i \delta \omega_j \rangle$ . After some mathematical manipulation, we can show that:

$$\langle \delta w_k \rangle = \sum_{r=2}^N x_{rk} \sum_{p,q} \frac{W_p}{W_q} (f_5 - 1) \frac{v_{pr} x_{q1}}{\mathbf{v}_k^T \mathbf{x}_k \lambda_1} \quad (2) \quad c_{ij} = \frac{1}{M \lambda_1^2} \sum_{k=2}^N x_{ki} \sum_{l=2}^N \frac{x_{lj}}{F_{kl}} E_{kl} \quad (3)$$

where  $E_{kl}$  is given by:

$$\begin{aligned} E_{kl} = & N^2 (f_7 - 2f_5 + 1) \sum_{I_1} W_z W_p \Omega_{zp}^{(kl)} + (2 - f_5 (f_6 + 1)) \sum_{I_2} G_{zn}^{(kl)} + N^2 (f_5 - 1)^2 \sum_{I_3} W_z W_p \Omega_{zp}^{(kl)} \\ & + N(1 - f_5) \sum_{I_4} W_p \Omega_{zqp}^{(kl)} + N(f_4 - 2f_5 + 1) \sum_{I_5} W_z^2 \Omega_{zz}^{(kl)} + N(f_1 f_5^2 - 2f_5 + 1) \sum_{I_6} W_m^2 \Omega_{zp}^{(kl)} \\ & + N(1 - f_5) \sum_{I_7} W_z \Omega_{znp}^{(kl)} \end{aligned} \quad (4)$$

Where  $M$  is the number of experts and  $1 \leq m \leq M$ .  $\Omega_{ij}^{(kl)} = v_{ik} v_{jl}$ ,  $\Omega_{jv}^{(kl)} = v_{ik} x_{j1} v_{vl}$ ,  $G_{ij}^{(kl)} = v_{ik} x_{j1} v_{jl} x_{i1}$ , while  $I_i$  is the set of quadruples  $(z, n, p, q)$  for which a)  $n=q$  and all other elements are distinct if for  $i=1$ , b)  $z=q$ ,  $n=p$  and  $z \neq n$  if  $i=2$ , c) all elements are distinct for  $i=3$ , d)  $z=q$  and the rest elements distinct for  $i=4$ , e)  $z=p$ ,  $n=q$  and  $z \neq n$  for  $i=5$ , f)  $z=p$  and while all other elements are distinct for  $i=6$  and finally g)  $n=p$  and all the other are distinct for  $i=7$ . Also we have  $F_{kl} = (\mathbf{v}_k \cdot \mathbf{x}_k^T)(\mathbf{v}_l \cdot \mathbf{x}_l^T)$ ,  $f_1 = 1 + s_w^2/12$ ,  $f_2 = 1 + s_w^2/12$ ,  $f_3 = (1 - 1/2 s_w)^{-1} (1 + 1/2 s_w)^{-1}$ ,  $f_4 = s_w^{-1} f_3 f_2 f_1$ ,  $f_7 = s_w \log f_3$ ,

$$f_5 = s_w^{-1} \log((1 + 0.5s_w)(1 - 0.5s_w)^{-1}) \quad (5) \quad f_6 = s^{-1} \log((1 + 0.5s)(1 - 0.5s))^{-1} \quad (6)$$

Equations (1)-(6) can be used to determine the mean values and the covariance matrix of the weight perturbations that can be used to calculate the MVNCDF which as previously discussed provides an estimate for  $P_{RR}$ .

In a similar way we can estimate the  $P_{RR}$  through the MVNCDF in the case where the GM method is used for the estimation of the weights from the PWC matrices. Taking into account the uncertainty model, the average weights  $w_k$  can be expressed:

$$w_k = W_k \left( \prod_{m=1}^M \prod_{j=1}^N \frac{\delta_{kj}^{(m)}}{W_j} \right)^{\frac{1}{MN}} \quad (7) \quad \delta_{kp} = \frac{1 + \Delta w_k^{(m)}}{1 + \Delta w_p^{(m)}} (1 + \Delta w_{kp}^{(m)}) \quad (8)$$

We define the perturbations  $\delta z_k^{(m)}$  of the logarithmic weights as  $\delta z_k^{(m)} = \ln w_k^{(m)} - \ln W_k$  and also define their successive differences  $v_k = \delta z_k - \delta z_{k-1}$  which follow a Gaussian distribution. The  $P_{RR}$  can be approximated from the MVNCDF once the covariance matrix and the mean values of  $v_k$  are determined. Taking into account that  $\langle v_k \rangle = \langle \delta z_k \rangle - \langle \delta z_{k-1} \rangle$  the covariance matrix is:

$$R_{\kappa\lambda} = \langle v_\kappa v_\lambda \rangle - \langle v_\kappa \rangle \langle v_\lambda \rangle = r_{\kappa\lambda} - r_{\kappa, \mu-1} - r_{\kappa-1, \mu} + r_{\kappa-1, \mu-1} \quad (9)$$

where  $r_{ij} = \langle \delta z_i \delta z_j \rangle - \langle \delta z_i \rangle \langle \delta z_j \rangle$  is the correlation matrix. After some mathematical manipulations we can show that:

$$\langle \delta z_k \rangle = \frac{1}{NM} \sum_{m=1}^M \sum_{j=1}^N \left\{ \langle \ln(1 + \Delta w_{kj}^{(m)}) \rangle - \ln W_j \right\} \quad (10)$$

$$r_{k\lambda} = \frac{1}{N^2 M^2} \left\{ \begin{array}{l} N^2 M Q_{k\lambda} + M \sum_{pq} Q_{kp\lambda q} \\ -NMf_7 + (NM-2)(N-1)f_9^2 \end{array} \right\} \quad (11)$$

where  $Q_{k\lambda} = f_7$  or  $Q_{k\lambda} = f_{10}$  for  $k=\lambda$  and  $k \neq \lambda$  respectively and  $Q_{kp\lambda q} = f_8 - f_9^2$  for the cases where a)  $k=\lambda=p=q$ , b)  $k=\lambda, p=q, p \neq k$ , c)  $k \neq \lambda, k=q, \lambda=p$ , otherwise is zero and:

$$f_7 = s_w^{-1} \left[ (0.5s_w + 1) (\ln^2(0.5s_w + 1) - 4) - (1 - 0.5s_w) (\ln^2(1 - 0.5s_w) - 4) \right] \quad (12)$$

$$f_8 = s^{-1} \left[ (0.5s + 1) (\ln^2(0.5s + 1) - 4) - (1 - 0.5s) (\ln^2(1 - 0.5s) - 4) \right] \quad (13)$$

$$f_9 = s^{-1} \left[ -s + (0.5s + 1) \ln(0.5s + 1) + (0.5s - 1) \ln(1 - 0.5s) \right] \quad (14)$$

$$f_{10} = s_w^{-1} \left[ -s_w + (0.5s_w + 1) \ln(0.5s_w + 1) + (0.5s_w - 1) \ln(1 - 0.5s_w) \right] \quad (15)$$

Equations (7)-(15) determine the covariance matrix and hence the MVNCDF can be used to provide an estimate for the  $P_{RR}$  in the case of GM method as well.

The above equations hold for linear or non-linear weights for both EV and GM methods.

Moreover, we describe how the proposed theoretical model for the estimation of  $P_{RR}$  can be used in practical situations, where the decision maker has no prior knowledge of the statistical parameters involved (i.e.  $W_k$ ,  $s$  and  $s_w$ ). Our method is based on the fact that these parameters can be inferred by the elements of the pairwise comparison matrices obtained by the experts. The estimates  $\tilde{s}$  and  $\tilde{s}_w$  for the perturbation strengths  $s$  and  $s_w$  are determined through the second central moments of the pairwise comparison elements. We can easily show that the sum of the moments of the upper diagonal elements is given by:

$$\sum_{i < j} \langle (P_{ij}^{(m)})^2 \rangle = s_w^{-1} f_1 f_2 f_3 \sum_{i < j} \frac{W_i^2}{W_j^2} \quad (16)$$

The parameters  $f_k$  are defined previously. The moments in the left hand-side can be approximated using the mean values  $I_{ij}$  of the squares of the actual elements provided

by the experts,  $\langle (P_{ij}^{(m)})^2 \rangle \cong \frac{1}{M} \sum_m (P_{ij}^{(m)})^2 = I_{ij}$ . To obtain the best possible estimates

$\tilde{s}$ ,  $\tilde{s}_w$  and  $\tilde{W}_k$  for the statistical parameters  $s$ ,  $s_w$  and  $W_k$  we could try least square fitting the approximation to the left hand side of (16) to its right hand side. This would lead to a constrained multi-variable minimization problem which may be hard to solve. We instead choose to minimize the following parameter,

$$Q(\tilde{s}, \tilde{s}_w) = \left[ \frac{1}{M} \sum_m \sum_{i < j} (P_{ij}^{(m)})^2 - \tilde{s}_w^{-1} \tilde{f}_1 \tilde{f}_2 \tilde{f}_3 \sum_{i < j} \frac{w_i^2}{w_j^2} \right]^2 \quad (17)$$

In (17), the parameters  $\tilde{f}_k$  are calculated from the equations provided for  $f_k$  in Section 2.5 by replacing  $s$  with  $\tilde{s}$  and  $s_w$  with  $\tilde{s}_w$ , i.e.  $\tilde{f}_1 = 1 + \tilde{s}_w^2 / 12$ ,  $\tilde{f}_2 = 1 + \tilde{s}^2 / 12$  and  $\tilde{f}_3 = (1 - \frac{1}{2} \tilde{s}_w)^{-1} - (1 + \frac{1}{2} \tilde{s}_w)^{-1}$  and we have also assumed that the average weights  $w_i$  provide a fair approximation for the original weights  $W_k$  (i.e.  $\tilde{W}_k \cong w_k$ ). This approximation simplifies the minimization and provides reasonably accurate results. The two-dimensional minimization of the function  $Q(\tilde{s}, \tilde{s}_w)$  can be performed using

standard minimization methods or exhaustive search and yields the estimates  $\tilde{s}$  and  $\tilde{s}_w$ . The estimated value  $\tilde{P}_{RR}$  of the probability of rank reversal can be determined in the same way as previously, where we apply the estimates  $\tilde{s}$  and  $\tilde{s}_w$  instead of the original statistical parameters  $s$  and  $s_w$  and use  $w_k$  as the as an approximation for the original weights  $W_k$ .

In this section, we have discussed a theoretical method for calculating the probability of rank reversal which quantifies the uncertainty of the outcome of pair wise comparisons. The method is based on the MVNCDF of the successive average weight differences. We have also theoretically calculated the mean value and cross correlation matrices of these differences that are needed in order to correctly use the MVNCDF. This was carried out both for the eigenvalue and the geometric mean methods of estimating the weights. The value of the theoretical method is twofold: first, it simplifies the estimation procedure, since we no longer need to rely on tedious and time-consuming Monte Carlo simulations used in numerical estimations of the probability of rank reversal. Second, much like any theoretical model, it can constitute a good starting point for further developing and extending the PWC framework. Our approach relies on a reasonable statistical uncertainty model that takes into account both the difference of opinions among the experts and the inconsistency in the completion of the pairwise comparison matrices. We have compared the results obtained through the MVNCDF, with those obtained through numerical simulations and a good agreement is observed, as representatively presented in Figure 4.

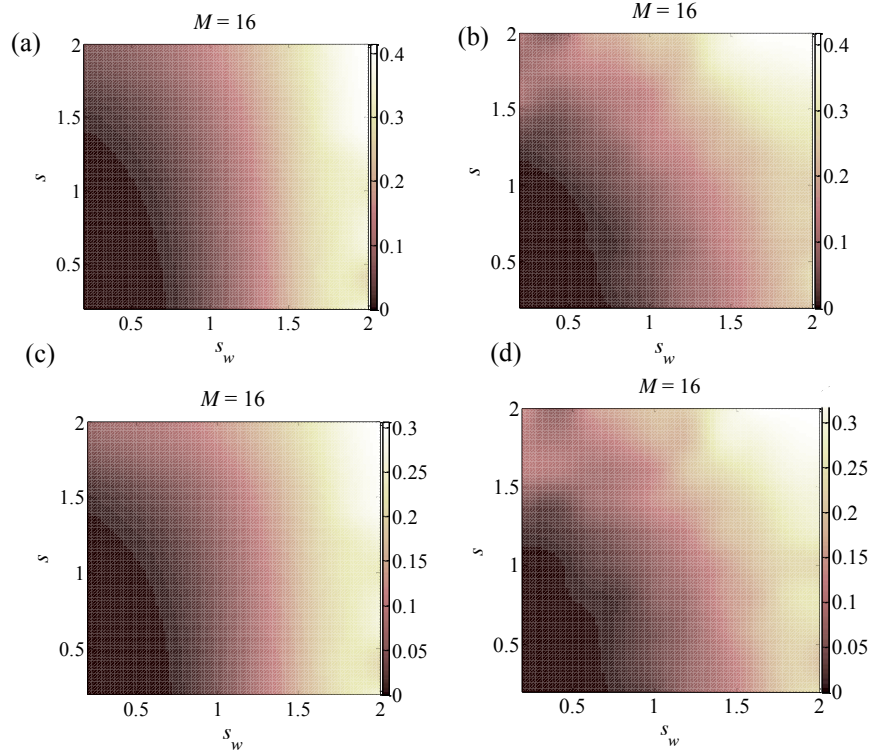


Figure 4.  $P_{RR}$  as a function of  $s$  and  $s_w$  for  $N=4$  criteria and  $M=16$  experts obtained by (a) the MVNCDF for EV, (b) MC simulations for EV, (c) the MVNCDF for GM, (d) MC simulations for GM

The results also show a slight advantage of the GM method over the eigenvalue method in terms of the probability of rank reversal. Finally, we discuss a procedure for estimating the probability of rank reversal in practice, where the statistical parameters

are unknown. We show how that these parameters can be estimated just from the pairwise comparison matrices of the experts and that the error in the probability of rank reversal is reasonable. The methodology presented in this chapter can be used to extend the pairwise comparison framework in order to provide some information on the credibility of the final outcomes of the decision making process.

### 3 Conclusions

In this thesis, an effective roadmap for the next generation home network has been developed. Based on pairwise comparison surveys, a number of technical, economic and social issues determining the penetration of future home networks have been also evaluated. Using the AHP the ranking of the various technological alternatives comprising of IEEE802.11, 60GHz systems and PLC has been accomplished. The results focus on network extension but are also indicative for the rest of the home network components as well. AHP indicates that PLC takes some precedence over the wireless alternatives. 802.11 is ranked second best while 60GHz system are regarded as a longer term alternative, which could provide gigabit per second connectivity. The thesis also discussed the merits of hybrid integration of these technologies, either in the PHY or a higher network layer. A sensitivity analysis was also performed in order to estimate the uncertainties involved and it was deduced that they do not in general undermine the results of the ranking.

In this thesis an evaluation of the potential of OW technologies for home/office network deployments was carried out. A number of important findings were obtained which must form part of any type of carefully designed roadmap for optical home networking technologies. The results suggest that a combination of VLC and IR hotspots, along with a PLC backbone provides the most favorable option. In any case, the results clearly indicated the advantages of PLC backbone in terms of ease of installation in older buildings. The ranking results were also further elaborated using sensitivity analysis and Monte Carlo simulation.

Considering the uncertainty issues that may undermine the decision making processes, we have applied an uncertainty model to address two important issues concerning the probability of rank reversal in pairwise comparisons. The first issue concerned how this probability is reduced by augmenting the number of experts participating in the surveys. The results dictate that there is not much to be gained by increasing the number of experts beyond 15, even if uncertainty level is large. The second issue concerned the problem of how the probability of rank reversal can be estimated in practice, from the elements of the pairwise comparison matrices of a single expert group. Two alternative methods have been discussed for extracting information on the statistical behavior of the uncertainty-induced perturbations and one of them provides reasonable good accuracy.

Finally, we have discussed a theoretical method for calculating the probability of rank reversal which quantifies the uncertainty of the outcome of pairwise comparisons. The method is based on the MVNCDF. We have compared the results obtained through the MVNCDF, with those obtained through numerical simulations and a good agreement is observed. Finally, we discussed a procedure for estimating the probability of rank reversal in practice from actual user data, where the statistical parameters are unknown.

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# Architectures for Dependable Modern Microprocessors

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**Abstract.** Technology scaling, extreme chip integration and the compelling requirement to diminish the time-to-market window, has rendered microprocessors more prone to design bugs and hardware faults. Microprocessor validation is grouped into the following categories, based on where they intervene in a microprocessor's lifecycle: (a) silicon debug: the first hardware prototypes are exhaustively validated, (b) manufacturing testing: the final quality control during massive production, and (c) in-field verification: runtime error detection techniques to guarantee correct operation. The contributions of this thesis are the following: (1) Silicon debug: We propose the employment of deconfigurable microprocessor architectures along with a technique to generate self-checking random test programs to avoid the simulation step and triage the redundant debug sessions, (2) Manufacturing testing: We propose a self-test optimization strategy for multithreaded, multicore microprocessors to speedup test program execution time and enhance the fault coverage of hard errors; and (3) In-field verification: We measure the effect of permanent faults performance components. Then, we propose a set of low-cost mechanisms for the detection, diagnosis and performance recovery in the front-end speculative structures. This thesis introduces various novel methodologies to address the validation challenges posed throughout the life-cycle of a chip.

**Keywords:** Dependability, silicon debug, testing, error, bug

## 1 Introduction

The evolution of semiconductor technology and computer architecture has radically transformed our world throughout the last decades. However, the combination of technology scaling and extreme chip integration, along with the compelling requirement to diminish the time-to-market window, has rendered microprocessors more prone to design bugs and hardware faults. The goal of this thesis is to provide solutions to the validation challenges posed from the microprocessor products throughout the life-cycle of a chip.

Microprocessor validation is grouped into the following categories, based on where they intervene in a microprocessor's lifecycle: (a) silicon debug: the first hardware

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<sup>1</sup> Dissertation Advisor: Dimitris Gizopoulos, Professor

prototypes are exhaustively validated, (b) manufacturing testing: the final quality control during massive production, and (c) in-field verification: runtime error detection techniques to guarantee correct operation. The contributions of this thesis are the following:

- **Silicon debug:** We propose the employment of deconfigurable microprocessor architectures along with a technique to generate self-checking random test programs to (a) avoid the time- and the resource-consuming simulation step, (b) triage the redundant debug sessions, and thus to accelerate silicon debug [2] [4].
- **Manufacturing testing:** We propose a self-test optimization strategy for multithreaded, multicore microprocessors to (a) speedup test program execution time, (b) enhance the fault coverage of hard errors, and thus to make manufacturing testing more efficient [1].
- **In-field verification:** We measure the effect of permanent faults performance components. Then, we propose a set of low-cost hardware-based mechanisms for the detection, diagnosis and performance recovery in the front-end speculative structures [5] [7] [10].

The share of silicon debug in the overall microprocessor chips development cycle is rapidly expanding. The validation step that detects the vast majority of design bugs is the one that stresses the silicon prototypes by applying huge numbers of random tests. Despite its bug detection capability, this step is constrained by the extreme computing needs for random test program simulation. Moreover, another major bottleneck and source of “noise” of this phase is that large numbers of random test programs fail due to the same or similar design bugs. This redundant behaviour adds long delays in the debug flow since each failing random program must be separately examined, although it does not usually bring new debug information. This thesis addresses both challenges of silicon debug. A self-checking methodology is proposed for generating random test programs (exploiting the ISA diversity property) that detect bugs by comparing the results of equivalent instructions combined with a technique to triage the failing test programs into categories with common failure modes. The proposed framework: (a) improves bug detection efficiency, (b) reduces the redundant debug session, and thus accelerates silicon debug.

When a sufficient level of coverage is reached the microprocessor design enters the production stage, where a last quality control is performed to detect any manufacturing defect. Functional self-testing forms an integral part of manufacturing test flow due to its at-speed testing and non-intrusive nature. Multithreaded (MT) SBST methodology proposes a novel self-test optimization strategy for multithreaded, multicore microprocessor architectures (OpenSPARC T1 microprocessor model). The proposed self-test program execution optimization aims to: (a) take maximum advantage of the available execution parallelism provided by multiple threads and multiple cores, (b) preserve the high fault coverage that single-thread execution provides for the processor components, and (c) enhance the fault coverage of the thread-specific control logic. MT-SBST methodology significantly speeds up self-test time, while at the same time it improves the overall fault coverage.

The combination of design complexity, shrinking time-to-market windows, and wear-out effects increases the failure probability of modern design and leads microprocessor manufactures to integrate numerous in-field verification mechanisms.



Trends such as low-voltage operation and process scaling are expected to significantly increase the rate of faults experienced by silicon. Their impact on a core's non-cache SRAM structures has not been accurately quantified. Faults in these structures will not affect correctness, but can cause severe performance degradation and variability among otherwise identical cores. We first classify and quantify the impact of permanent faults in the performance components of modern microprocessors. Then, we propose a low-cost microarchitectural mechanism that exploits the self-verification property of predictors to achieve performance recovery.

This thesis introduces various novel methodologies to address the validation challenges posed throughout the life-cycle of a chip. The proposed techniques make the validation process more efficient and are easily applicable to the existing industrial flow.

## **2 Silicon debug**

Aggressive technology scaling and extreme chip integration, combined with the compelling requirement to diminish the time-to-market window have rendered microprocessors more prone to design bugs than ever. As a result, silicon debug – the process of validating and debugging a new microprocessor design on its first silicon prototype chips – has evolved to a critical, time-consuming, and labour-demanding step in a chip's development flow [11]. Recent trends [16] show that the time spent from the arrival of the first silicon prototype chip to high volume production ramping up is steadily growing, while the ratio between the size of the design and the debug teams has reached 2:1. Thus, an efficient silicon debug approach that promptly detects and eliminates the design bugs before volume production can make the difference between success and failure of a microprocessor product.

Silicon debug starts with the arrival of the first prototypes and often continues well after a product has gone to volume production. A comprehensive suite of test programs covering many test scenarios are executed on the prototype chips to detect bugs that can be anything from logic/functional bugs, electrical or process-related bugs to mask-related manufacturing defects [14]. Subsequently, for each failing test program (one that does not execute correctly due to a bug), separately, a systematic debug phase is performed by the debug engineers to identify the root cause of the failure.

Massive application of automatically generated random test programs on the prototype microprocessor chips is one of the most effective parts of silicon debug [13]. Despite its bug detection efficiency, this step is constrained by extreme computing needs for random tests simulation to extract the bug-free memory image for comparison with the actual silicon image. Another major bottleneck and source of “noise” in this phase is that large number of random test programs fail due to the same or similar design bugs. This redundant behaviour prolongs silicon debug phase since each failing random test program must be exclusively root-cause analysed, although it does not usually bring new debug information. Finally, volume production may be further prolonged due to bugs that lurk behind other bugs. These blocking

bugs stall the execution of the subsequent tests, since no workaround exists and therefore additional re-spins are needed.

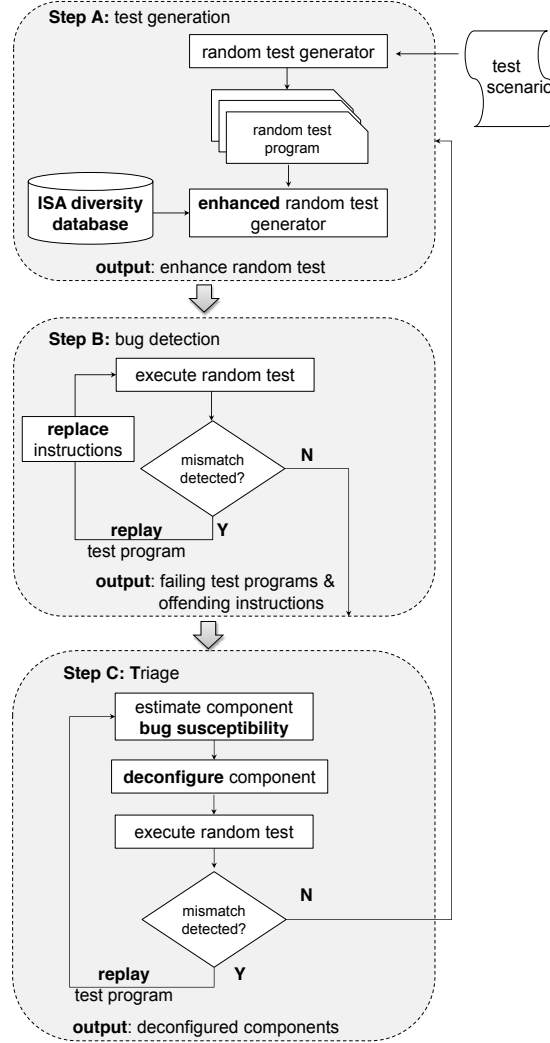
This work introduces a silicon debug methodology for microprocessors with two major objectives: (a) increase coverage by applying more tests to silicon prototypes; and (b) reduce validation time by triaging the redundant failing random test programs. The methodology does so by exploiting (1) the inherent diversity of microprocessor instruction sets to eliminate the time consuming simulation step by employing self-checking tests; and (2) the property that allows hardware components to be deconfigured without compromising microprocessor's functional completeness to bucketing the redundant failing test programs. Figure 1 shows an overview of the flow.

**A. Test generation:** The fundamental first step is the identification of ISA diversities, i.e. microprocessor instruction equivalences, and the population of the ISA diversity database. The database contains for each instruction a list of equivalent instruction sequences. Then, the flow is fed with the random test programs (original RiTs) already generated (but not simulated) by sophisticated random test program generators that all microprocessor manufactures internally use [11] [12]. We pair each original RiT with an Equivalent RiT to generate an enhanced RiT. An eRiT is automatically generated from an original RiT replacing its instructions with their equivalent counterparts that have been stored in the ISA diversity database. Finally, a checking code compares the stored results of the original RiT and the eRiT to identify mismatches. A mismatch indicates a potential silicon bug.

**B. Bug detection:** Combining the self-checking method, with a hardware replay mechanism (Figure 2– right part) enables the extraction of as much as possible useful debugging information regarding the bug detection capability of each test program and provides a fast workaround solution to bypass blocking bugs. The hardware mechanism records the failing comparisons when mismatches are detected and replays the execution of the original RiT by replacing the execution of the offending instruction with its equivalent. In particular, the “replacement” is done on-the-fly using the program counter of the store instructions saved in buffers store-addr and estore-addr. During the first run of the enhanced RiT, the checking code finishes with the mismatches between the set of  $k$  responses of the original RiT and the eRiT stored in `mids-queue` (mismatch id queue), with `mid` between 0 and  $k$ . If `mid` = 0 (i.e. the queue is empty), then there is no mismatch and the chip passes the enhanced RiT; debug continues with the next RiT.

If queue is not empty (i.e `mid` > 0) the enhanced RiT will be replayed `mid` times, because `store[mid]` and `estore[mid]` instructions generated different results (Figure 3 – instr.16). The key functionality of the mechanism is that when a mismatch is detected between `store[i]` and `estore[i]`, during replay, instead of executing the “buggy” code between `store[i-1]` and `store[i]`, the processor executes the equivalent code between `estore[i-1]` and `estore[i]`. The mismatch has been bypassed, subsequent responses are not corrupted and if the remaining test can detect another mismatch (more bugs) it is allowed to do so. A list of mismatch identifiers (`mids`) is the log information our method provides. An integer `m` in the log (an entry in the `mids-queue`) means that: (a) the  $m$ th pair of stores produced a mismatch, i.e. `store[m]` and `estore[m]` produced different results; (b) the code between `store[m-1]` and `store[m]` has been replaced by the code between `estore[m-1]` and `estore[m]` and the original RiT continued. These

two pieces of information can help the debug engineer identify the offending instructions and work on them.



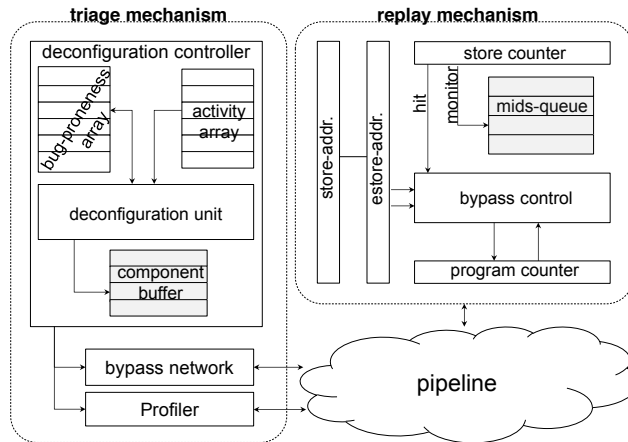
**Figure 1:** The proposed silicon debug flow.

**C. Triage:** As soon as bug detection phase finishes, hardware-assisted triage begins. Hardware triage is assisted through the integration of the triage mechanism (Figure 2–left part). For each failing self-checking random test, the triage mechanism selects the component that is most susceptible to contain a bug and deconfigures it in the next execution of the failing random test program. This process is repeated until the test program is correctly executed (i.e. the bug has been “masked” by the sequence of deconfigurations). All test programs that eventually execute correctly after the same sequence of deconfigurations are grouped into the same “bucket”. Intuitively, the bug

that causes the failure most probably resides within the components that have been deconfigured before the test executes correctly.

The outcome of this step is a list of components that have been deconfigured and is stored in the component buffer (each entry of this array saves the id of the component). The interpretation of the list provides the following triage-related information: (a) Empty list. The random test program was correctly executed. No failure detected; no debug action required in the morning, (b) List contains a set of the deconfigurable components. The random test program was correctly executed after components {Ck, Cn, Cm, Cq} have been deconfigured. The list of components indicates a “bucket” of failing test programs. All test programs ending with the same list of deconfigurations are grouped together; and (c) List contains all deconfigurable components. The random test program fails even after all deconfigurable components are turned off. No triage grouping information; the random test must be separately debugged.

At the end of the multiple hardware-enabled test program re-executions, the contents of the component buffer and the mismatch identifier queue (grey colored boxes in Figure 2) are downloaded along with the remaining memory image of the prototype on the host machine (i.e. dedicated server that controls the entire validation campaign) for further analysis by the debug engineers. It should be noted that the proposed methodology detects bugs (both logical and electrical) with the following characteristics: (i) their excitation does not depend on the operational conditions (temperature, voltage, frequency); and (ii) they continue to manifest themselves despite the deconfiguration of components from the overall design.

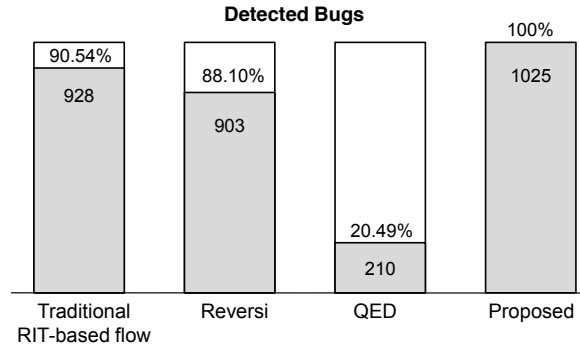


**Figure 2:** The proposed hardware mechanism for silicon debug acceleration.

To evaluation the proposed silicon debug methodology, we set up the tool chain on top of the PTLsim [18] architectural simulator as presented in [4].

**First set of experiments:** We compare our methodology, in terms of bug efficiency, with the traditional flow (mismatches are only detected off-line comparing the memory dumps of the actual execution with the expected memory dump contents from simulation) and with two other self-checking validation approaches [15] [17]. For each of the three methods, we use the same original RiT (4K instructions) as input and we enhance it according to the basic idea of each method. Our methodology

detects all 1K bugs injected into the simulator (Figure 3) because we stopped generation of more RiTs when all the injected bugs were detected. The traditional flow detects 928 bugs (coverage 90.54%). This difference, against the proposed method, is explained by the activation of more hardware areas by the equivalent RiT. The approach of [17] detects 903 bugs (coverage 88.10%) because there are cases where an instruction cannot be reversed. Furthermore, the flexibility of the ISA diversity concept to deploy equivalent instructions which activate totally different paths in processor’s logic provides us with the ability to avoid bug masking conditions. Finally, [15] detects 210 bugs (coverage 20.49%) because it can only detect electrical bugs, since a logic bug will act in an identical way in both original and duplicated instruction. For a complete silicon debug plan (trillions of instructions), we expect our approach to have the same bug efficiency as the traditional flow since our bug detection capability relies on the original RiTs which are carefully generated by sophisticated industrial random generators. The advantage of our method is that by avoiding the time-consuming simulation step it is able to apply many more RiTs and thus detect potential bugs much earlier.



**Figure 3:** Design bug coverage for the four different methods.

**Second set of experiments:** The proposed method refines the debug information using the hardware replay mechanism. During our bug injection experiments, we observed that the average number of different bugs that were detected by a single RiT is about 4. To verify the effectiveness of our approach on refining debug information, we activate the hardware replay mechanism in our infrastructure (the triage mechanism is disabled) and conducted a second set of experiments: we injected all the bugs at the beginning of the simulation and executed all RiTs with the highest bug detection capability. The proposed hardware mechanism detected all the injected bugs (through bypassing the offending instructions with their equivalents in the replay executions). This is a significant benefit of the proposed framework compared to the traditional flow which requires more tests to detect the same number of bugs.

**Third set of experiments:** To demonstrate the benefits of the triage mechanism on test program triaging, we have selected a set of 10 hard-to-detect logic bugs from the set of injected bugs distributed among the deconfigurable modules of PTLsim simulator (we characterize them as hard-to-detect because all 10 bugs are detected by a small number of test programs; smaller than the average case). Furthermore, all 10 design bugs are together injected from the beginning of the bug injection campaign,

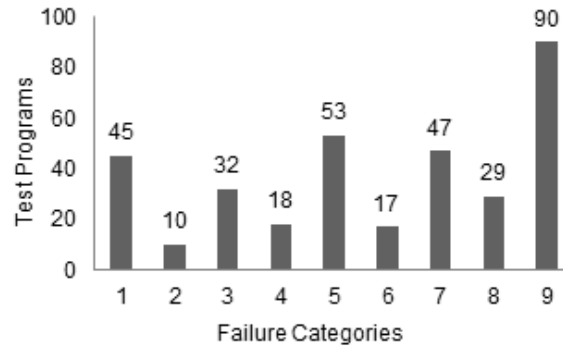
as an attempt to model more accurately the silicon debug environment where all bugs can co-exist in the prototype chip. We repeated the experiments only for a subset of the initial random test programs that are affected from them; these are 341 test programs.

Table 1 presents details about the selected design bugs. The first column is the id of each bug, while the second column gives the microprocessor component in which the bug resides. Issue Queue<sub>1</sub> and Issue Queue<sub>2</sub> refer to different components in the microprocessor design (Issue Queue<sub>1</sub> for the integer cluster, and Issue Queue<sub>2</sub> for the floating point cluster). The third column shows the number of test programs affected by each design bug when injected individually (from the first set of experiments) and the last column provides a short description.

Bug ID	Component	Failing Test Programs	Bug Description
1	Conditional Predictor	45	Update fetch address on branch misprediction fails
2	RAS	10	Incorrect push to stack
3	Issue Queue <sub>1</sub>	32	Dependent uop issued, while producer is waiting in ready-to-write-back state
4	Issue Queue <sub>2</sub>	21	Entry not flushed on a branch misprediction
5	Floating Point Unit	50	Incorrect rounding operation
6	Data cache	17	Valid array logic; invalid data read
7	Load Queue	47	Load to store aliasing
8	Store Queue	29	Store data before address gets valid
9	Reorder Buffer	48	Commit entry more than once
10	Reorder Buffer	42	Invalid control bit activation
<b>Total</b>		<b>341</b>	-

**Table 1:** Details of 10 hard-to-detect design bugs.

Figure 4 shows the results for this set of experiments. The horizontal axis presents the different “buckets” of failing random test programs that are formed when the proposed methodology is applied. The vertical axis shows the number of failing test programs of each bucket.



**Figure 4:** Failure categories for the 341 failing test programs.

The application of the proposed methodology with the deconfiguration mechanisms enabled results in a triaging of the 341 random test programs in 9 different failure categories shown in Figure 4:

- Failure categories 1, 2, 3, 4, 6, 7, and 8 group the test programs that are affected exclusively from the design bugs in one of the following microprocessor components: Conditional Predictor, RAS, Issue Queue<sub>1</sub>, Issue Queue<sub>2</sub>, Data Cache, Load and Store Queues, respectively. As a result, when the deconfiguration controller turned the corresponding microprocessor component off, the bug is “masked” and the test program execution is correct.
- Failure category 5 groups 53 random test programs, while the expected number of test programs affected from a design bug in the FPU unit is 50. The reason for that is that these particular test programs (3 from Issue Queue<sub>2</sub>) were able to detect more than one design bugs (design bugs injected both in the Issue Queue<sub>2</sub> and the FPU). As a result, only when both buggy microprocessor components were deconfigured the re-execution of the test program results in a correct execution.
- Failure category 9 includes the test programs that fail due to bugs 9 and 10 injected in the Reorder Buffer’s logic. The deconfiguration mechanisms were unable to distinguish these design bugs into different categories, since both of them were inside the deconfiguration granularity of the ROB structure. Specifically, these bugs reside in neighboring entries of the re-order buffer and manifest themselves as invalid dependency re-dispatching when a mis-speculation happens. Therefore, the same sequence of deconfiguration results in masking both bugs..

Clearly, the proposed flow has a profound impact on the effectiveness of silicon debug and greatly accelerates root cause analysis by removing the “noise” of redundant random tests that fail due to the same underlying bug (the 341 initial debug sessions are reduced to only 9 in the last set of experiments).

### 3 Conclusions

Today, the pervasiveness of microprocessors, the most complex and immensely powerful application of electronics, in our society goes far beyond the wildest imagination. The same path that is leading technologies toward these remarkable achievements is also making them increasingly unreliable posing a threat to our society. Silicon technology process scaling trends, modern architecture complexity and the compelling requirement to diminish the Time-to-Market threaten to create a “validation wall”. As a result, semiconductor industry and academic researchers must explore radical solution and develop innovative techniques to address the dependability challenges of the current and the forthcoming microprocessors. This thesis introduced novel methodologies to address the validation challenges posed throughout the life-cycle of a microprocessor.

Microprocessor validation is grouped into three categories, based on where they intervene in a microprocessor’s lifecycle: (a) silicon debug: the first hardware prototypes are exhaustively validated, (b) manufacturing testing: the final quality control during massive production, and (c) in-field verification: runtime error detection techniques to guarantee correct operation. This thesis introduces various

techniques to tackle the challenges of microprocessor validation targeting to: (a) make the dependability process more efficient; and (b) be easily applicable to the existing industrial flow. The contributions of this thesis are as follows:

- **Silicon debug:** The share of silicon debug in the overall microprocessor chips development cycle is rapidly expanding due to the ever growing design complexity and the limited throughput of pre-silicon verification methods. Massive application of short random test programs on the prototype microprocessor chips is one of the most effective parts of silicon debug. Despite its bug detection capability, it is constrained by extreme computing needs for random test programs simulation to extract the bug-free memory image. Another major bottleneck and source of “noise” in this phase is that large numbers of random test programs fail due to the same or similar design bugs. This redundant behavior adds long delays in the debug flow since each failing random test program must be separately examined, although it does not usually bring new debug information. We proposed the employment of self-checking random test programs along with a deconfigurable microprocessor architecture to avoid the time-consuming simulation step, triage the redundant debug sessions and thus accelerate silicon debug. To do so, we exploited the inherent diversity found in all popular Instruction Set Architectures (ISAs) and the ability to deconfigure hardware modules without affecting the functional completeness of a design. Detailed evaluation of the method on an x86 microprocessor model demonstrated its effectiveness in accelerating silicon debug.
- **Manufacturing testing:** We presented an efficient multithreaded (MT) SBST methodology that optimizes self-test time taking maximum advantage of thread-level parallelism while at the same time enhances the self-test program error detection capability on the thread-specific control logic of the processor. The methodology contributed to the effective application of SBST in manufacturing testing. Our experiments on OpenSPARC T1 revealed that the proposed methodology improved significant test execution time at both the core level (3.6 times) and the processor level (6.0 times) against single-threaded execution, while at the same time it improves fault coverage. Compared with a straightforward multithreading approach, it reduces the self-test time at both the core level and the processor level by 33% and 20%, respectively. Overall, our methodology guarantees high stuck-at fault coverage (88% for the entire processor, more than 1.5M logic gates), which is the highest coverage ever reported in the literature by a software-based functional test methodology in such a complex industrial microprocessor.
- **In-filed verification:** Aggressive technology scaling along with low voltage operation exacerbates the likelihood and rate of hard faults not only in large SRAM arrays (such as cache memories), but also in non-SRAM microprocessor structures. Some of the largest non-cache SRAM structures support speculation such as the branch predictor tables, the branch target buffers, and the data prefetcher. Faults in these structures will not affect correctness, but can cause severe performance degradation and variability among otherwise identical cores. We accurately classified and quantified the performance impact of hard faults in non-SRAM structures over a set of CPU benchmarks. To do so, we applied a statistically safe fault injection campaign for single and multiple faults a modified version of the cycle-accurate x86 architectural simulator PTLsim running the SPEC CPU2006 suite. Our evaluation revealed significant differences in the effect of faults and their performance impacts



across the components as well as within each component. In particular, we demonstrated that a very large fraction (44% to 96%) of hard faults in these components leads to performance fluctuation. Furthermore, faults in the data prefetcher degrade IPC by up to 26%, compared to fault-free operation, while faults on the branch prediction unit reduce IPC by more than 16%, respectively. Moreover, we found that faults in these components can substantially increase the performance variability across identical cores. Finally, we proposed low-cost microarchitectural techniques to diagnose predictor faults and recover the performance loss. Our techniques exploited the self-verification property of predictors to achieve performance recovery at lower cost than comparable techniques. We found that our solutions can recover almost all performance loss and virtually eliminate performance variability among cores.

The research outcomes of this thesis open the door to several future directions. Future systems architectures must be designed to facilitate hardware validation. In particular, future solutions should have adhered to the following guideline principles: (a) low-power, (b) negligible area overhead, (c) scale with design complexity; and (d) highly automated. In the silicon debug domain, future research should focus on the automation and standardization of the design bug detection and root-cause analysis process. Furthermore, this thesis demonstrated the effectiveness of software-based techniques in accelerating manufacturing testing and guaranteeing a high level of fault coverage. This may be an indication that future microprocessors should devote valuable silicon estate in hardware hooks that enable the at-speed, low-cost testing. The growing demand for high-performance computer systems pushes computer architects to integrate numerous performance mechanisms in the microprocessor designs. However, functional correctness is prioritized over performance correctness. This work revealed that faults in performance components can lead to noticeable performance loss and variability in otherwise identical cores. Therefore, future designs must integrate mechanisms to continuously monitor the system performance health and applying contingency actions. Finally, a vital future research direction is to bridge the gap between silicon debug, manufacturing testing and in-field verification techniques through the development of cross-cutting solution that will operate throughout the entire life-cycle of a microprocessor.

The vital challenge of future technologies is to build dependable systems. This thesis proposed various novel techniques to make the validation process, throughout microprocessor life-cycle, more effective in terms of bug/error detection efficiency, resource- and time-budget. We hope that the contributions presented in this thesis will advance the research in manufacturing dependable microprocessor architectures and will find applicability in future commercial microprocessor products.

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# Infinite-game Semantics for Logic Programming Languages

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**Abstract.** This thesis focuses on the study of the semantics of logic programs and the development of infinite games of perfect information between two players, that capture this semantics. We define a game that, given a propositional logic program with negation and a ground atom that belongs to it, may have three possible results (win for any of the players or tie). The game is determined. Based on this, we get a game interpretation of the program that is a model. Moreover, it is equivalent to the well-founded semantics of the program. In order to prove that, we use a new refined game that has infinite possible outcomes. The refined game is also proved to be determined and its game interpretation is a model of the program and equivalent to the infinite valued minimum model semantics of the program. Our study then extends to intensional logic programming a generalization of temporal and modal logic programming. For the monotonic case we develop a game semantics equivalent to the existing semantics. More importantly, we extend it to a three-valued game semantics, the first semantics for the broad class of non-monotonic intensional logic programming languages.

## 1 Dissertation summary

The purpose of this dissertation is to use infinite games in order to provide an alternative, simple and elegant semantics for logic programming languages. We consider two such languages and provide for each one of them a corresponding infinite game that captures its semantics. The first language is that of normal logic programs, i.e. logic programs that allow negative literals in the bodies of clauses. The second language is intensional logic programming, i.e. logic programming extended with intensional operators.

The semantics of logic programming has been extensively investigated. Probably the most broadly studied topic in the area is the problem of extending logic programming with negation. The generally accepted computational interpretation of negated atoms is *negation-as-failure*. Intuitively, a goal  $\sim A$  succeeds iff the subcomputation which attempts to establish  $A$  terminates and fails. After many years of research, it appears that the most widely acceptable approaches to the semantics of negation-as-failure are the *well-founded semantics* [35] and

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the *stable model semantics* [14]. The former approach provides a unique “distinguished” model of the program while the latter allows for the possibility of zero, one or many models. This thesis focuses on the well-founded semantics.

Game-theoretical semantics has been extensively studied in logic and language [15]. Despite the fact that game semantics is well-established for more mainstream programming languages [2], their application to logic programming has been very restricted. To our knowledge, there exist two other works that deal with the problem of giving a game semantics to logic programming. However, both of them deal with the negation-free case. The first of them, appears in [34] in which M. H. van Emden develops a probabilistic version of logic programming whose proof theory is described using a two-person game. This work, although ground-breaking, does not treat negation. More recently, the game for the negation-free case was also studied in [6], and interesting connections with the classical semantics of logic programming have been established. Less directly connected to our work but very indicative of the connections between game theory and logic programming, is the work of M. De Vos (see for example [5], [4]). More specifically, in [4] certain new logic programming formalisms are introduced in order to model decision-making. It is demonstrated that strategic games and extensive games of perfect information can be represented in these new formalisms in such a way that the equilibria of the games can be retrieved as the stable models or answer-sets of the programs.

The starting point of our investigation is the simple game semantics for ordinary negation-less logic programming in [34]. Suppose we have a program  $P$  and a goal clause  $G$ . We describe how the question, “does  $G$  succeed as a query to  $P$ ” can be reduced to the question, “does Player I have a winning strategy in the game  $\Gamma_{PG}$ ”. The game  $\Gamma_{PG}$  is a two person infinite game of perfect information. Player I, who we will also call the Believer, believes that  $G$  will succeed and his first move is to play  $G$ , thus asserting his belief. Player II, who we will also call the Doubter, thinks  $G$  will fail. His first move is to choose one of the variables in  $G$  which he thinks will fail on its own, and plays it, thus asserting his doubts. From then on, the play proceeds as follows: the Believer (who thinks the variable just played by the Doubter will in fact succeed) must play a clause in the program whose head is the variable just played; and the Doubter must, on his turn, play one of the variables in the body of this clause.

Either player can win by making a move for which his opponent has no legal response. For the Believer, this means playing a clause with an empty body; this happens when the Doubter chooses to doubt an atom for which there is a fact in the program. For the Doubter, this means choosing a variable for which there is no rule; in this case the Believer has chosen a rule with a variable in its body for which there is no evidence. Finally, we must give the Doubter an important advantage: he wins if the game never ends.

It is not hard to argue informally about the correctness of the game semantics for negation-less programs. If  $G$  actually fails, the Doubter’s winning strategy is to repeatedly choose variables which themselves fail. If  $G$  succeeds, the Believer’s winning strategy is to repeatedly choose rules that are applicable, ie., for which

all the variables in the body succeed. The only subtle point is that the Believer, in choosing applicable rules, must avoid ones like  $p \leftarrow p$  which do not actually advance the game.

Once the standard game is understood in terms of the informal anthropomorphic description given above, it is not hard to see how to extend it to programs with negation. There is one new rule: when one of the players plays a formula of the form  $\sim p$ , his opponent *must*, on the next move, play  $p$ . And this move must then be answered by playing a clause whose head is  $p$ , and so on. The significance of the new rule is that when a negation is encountered, the players swap roles - the Believer becomes the Doubter and vice-versa. For example, suppose that Player II, who doubts  $q$ , has just played it. Player I, who believes  $q$ , plays the clause  $q \leftarrow r, \sim p$ . Then Player II, who doubted  $q$ , thinks the weak link is  $\sim p$ , and plays it. Player I, who believes  $q$ , must believe  $\sim p$ , which means doubting  $p$ , and playing it. Thus Player I, who was a believer and believed in  $q$ , has now become a doubter, who doubts  $p$ . His opponent, who was a doubter, is now a believer (in  $p$ ) and must find a rule for  $p$  to play.

The rules for winning or losing require modification. As before, any player who has no legal move loses immediately. Thus either Player I or II can lose if they find themselves, in the doubter's role, doubting a fact or, in the believer's role, believing without evidence. Furthermore, if the game play is infinite and after a certain point one of the players remains a doubter, he wins. Finally, if the players swap roles infinitely often, the result is a tie.

The game we have just described is determined, i.e. it always has a value, and equivalent to the well-founded semantics of negation. The well-founded semantics is based on a three-valued logic, namely a logic that uses the truth values *False*, 0 and *True*. Intuitively, we need to demonstrate that an atom has value *True* (respectively *False*) in the well-founded model iff Player I (respectively Player II) has a winning strategy in the corresponding game. Additionally, we have to show that the value 0 corresponds to the case where the best choice for both players is to lead the game to a tie. Establishing the equivalence that we just described is not straightforward. The reason is that, as we are going to see, the well-founded model is constructed in stages, and the truth values that are introduced in different stages can be thought of as having different "strengths". On the other hand, the game we have described does not have any notion of different levels of winning or losing. Therefore, in order to establish the equivalence it would be convenient if we had on the one hand a refinement of the well-founded model in which the strengths of truth values are as explicit as possible and on the other hand a refinement of the game that uses different degrees of winning and losing.

A characterization of the well-founded model that captures in a logical way this notion of different strengths of truth values has been introduced by P. Rondogiannis and W. W. Wadge in [29, 28]. More specifically, the *infinite-valued semantics* introduced in [29, 28] is a refinement of the well-founded semantics and it uses instead an infinite number of truth values ordered as follows:

$$F_0 < F_1 < F_2 < \dots < 0 < \dots < T_2 < T_1 < T_0$$

Inspired by this semantics, we define a refined game which supports different degrees of winning and losing. We show that this game is also determined. We then demonstrate that the interpretation of the program that we get using this new infinite-valued game, is equivalent to the infinite-valued semantics. This will immediately imply that the game interpretation of the initial three-valued game is equivalent to the well-founded semantics.

Our study then focuses on *Intensional logic programming*, an extension of logic programming based on intensional logic. *Intensional Logic* is an extension of classical logic that was introduced by R. Montague [17] in order to capture the semantics of natural languages. Roughly speaking, intensional logic was proposed as a formal system for understanding and reasoning about *context-dependent* properties of natural language expressions. In its initial form, intensional logic was a higher-order one, equipped with modal and temporal operators [13]. However, the term “intensional logics” can also be used more loosely in order to describe a large class of logics for reasoning about context-dependent phenomena [20]. *Temporal logics* and *modal logics* are special cases of intensional logic.

Based on this broad interpretation of the term, M. Orgun and W. W. Wadge introduced in [24] the notion of *intensional logic programming*, which includes as special cases many non-classical extensions of logic programming (such as *temporal logic programming*, *modal logic programming*, and so on). As pointed out in [24], numerous logic programming languages that have been proposed in the literature can be characterized as “intensional” (such as Chronolog [24], Tempura [19], Molog [7], Cactus [27], MProlog [22] and so on). It was therefore natural to wonder whether there exists a common semantic framework for handling all these systems in a uniform way. As it was demonstrated in [24], if the intensional operators of the source intensional logic programming language obey some simple semantic properties, then the programs of the language are guaranteed to possess the *minimum model* property. However, all the intensional operators allowed in [24] are assumed to satisfy the *monotonicity* property and this excludes many interesting applications that involve *non-monotonicity*, which is a crucial concept involved in knowledge representation and reasoning.

Our purpose is to extend the framework of [24] to allow arbitrary (non-monotonic) intensional operators in the bodies of program clauses and to define a general semantic framework for non-monotonic intensional logic programming. Our approach is again based on *game semantics*.

We begin by constructing a simple two-person game for the class of intensional logic programs considered in [24] (in which the intensional operators are monotonic, universal and conjunctive). We demonstrate that the outcome of the game coincides with the minimum model semantics obtained in [24]. In this way we provide an equivalent formulation to the approach of Orgun and Wadge for facilitating the further study of intensional logic programs.

We then extend the proposed game to handle intensional logic programs that even use non-monotonic operators in the bodies of clauses and show that the games are determined. In this way we obtain the first general semantic framework for non-monotonic intensional logic programming. It should be noted

that intensional logic programming, due to its variety of operators, allows a much broader framework for non-monotonicity than classical logic programming where the main source of non-monotonicity is the operator of negation-as-failure.

## 2 Main results

### Game Semantics of Normal Logic Programming

Let  $P$  be a logic program and  $G$  a goal clause. We define a corresponding PI-game  $\Gamma_{PG} = (X, T_\omega, D, \Phi)$ , as follows: The set of moves  $X$  is:

$$X = \{G\} \cup P \cup \text{literals}(P_G) \cup \text{negvars}(P) \cup \langle I've \text{ lost} \rangle \cup \langle I've \text{ won} \rangle$$

In other words, a player can choose one of the following moves: a) he can play the goal clause, or b) play a clause of the program, or c) a literal that appears in  $G$  or in the body of a clause of  $P$ , d) a propositional variable that appears in a negative literal in the body of some clause of  $p$ , or finally, declare losing or winning.

We can now specify the rules that the two players must obey:

- (R1) The first move of Player I is the goal clause  $G$  and the next move, by Player II, is the literal in the body of the goal clause.
- (R2) If the previous move is a clause (with non-empty body), the next move is one of the literals in the body of the clause.
- (R3) If the previous move is a positive literal  $p$ , the next move is a clause in  $P$  whose head is  $p$  (and whose body could possibly be empty).
- (R4) If the previous rule is a negative literal  $\sim p$ , the next move must be  $p$  itself (this last move is called a **role-switch**).
- (R5-6) If none of the above rules is applicable, the player breaks the rules and loses (plays the  $\langle I've \text{ lost} \rangle$  move from then on) while the other player wins (plays the  $\langle I've \text{ won} \rangle$  move from then on).

Notice that if in rule (R2) the body of the clause is empty, then we will say that the player is *forced to break rule (R2)*. Similarly, the player *is forced to break rule (R3)* if he can not find a clause in  $P$  whose head is  $p$ . We should note here that since our game is infinite, a play continues even after one of the two players has broken the rules and the game has essentially ended in favor of one of the two players. The player who is forced to break the rules keeps on playing the move  $\langle I've \text{ lost} \rangle$  while the other, who has won the play, keeps on playing the move  $\langle I've \text{ won} \rangle$ . However, the moves beyond this point will be irrelevant to the outcome of the play. This way every play is infinite. A play that does not contain  $\langle I've \text{ won} \rangle$  and  $\langle I've \text{ lost} \rangle$  moves will be called a *genuinely infinite play*.

More formally, the infinite tree  $T_\omega$  of the game  $\Gamma_{PG}$  consists of all infinite sequences  $\langle x_0, x_1, \dots, x_k, \dots \rangle$ , which satisfy the following restrictions:

- R<sub>1</sub>**:  $x_0 = \langle \leftarrow p \rangle$ , where  $G = \leftarrow p$  is a goal clause, and  $x_1 = \langle p \rangle$ .
- R<sub>2</sub>**: If  $x_k = \langle q \leftarrow q_1, \dots, q_n \rangle$ , then  $x_{k+1} = \langle q_i \rangle$ , where  $0 < i \leq n$ .
- R<sub>3</sub>**: If  $x_k = \langle p \rangle$ , then  $x_{k+1} = \langle \mathcal{C} \rangle$ , where  $\mathcal{C}$  is a clause whose head is  $p$ .

- R<sub>4</sub>**: If  $x_k = \langle \sim p \rangle$ , then  $x_{k+1} = \langle p \rangle$  .  
**R<sub>5</sub>**: If after  $x_k$  has been played none of the above rules is applicable, then  $x_{k+1} = \langle I've\ lost \rangle$ .  
**R<sub>6</sub>**: If  $x_k = \langle I've\ lost \rangle$ , then  $x_{k+1} = \langle I've\ won \rangle$  (and vice-versa).

The set  $D$  of rewards is the set  $\{F, 0, T\}$ . Intuitively,  $F$  corresponds to the *False* truth value,  $T$  to the *True* truth value and 0 to an intermediate truth value that is above *False* and below *True*. From the game point of view,  $F$  corresponds to a win of Player II,  $T$  to a win of Player I, and 0 to a tie of the two Players.

Let  $a \in \text{Strat}^I(\Gamma)$  and  $b \in \text{Strat}^{II}(\Gamma)$  be two strategies, and let  $s = a \star b$  be the unique play determined by  $a$  and  $b$ . The following two definitions will be useful in defining the payoff function:

**Definition 1.** Let  $P$  be a program,  $G$  a goal, and let  $s$  be a play of the corresponding game  $\Gamma_{P_G}$ . Then,  $s$  is called a *true-play* if either Player II plays the  $\langle I've\ lost \rangle$  move in  $s$  or if  $s$  is a genuinely infinite play that contains an odd number of negative literals.

**Definition 2.** Let  $P$  be a program,  $G$  a goal, and let  $s$  be a play of the corresponding game  $\Gamma_{P_G}$ . Then,  $s$  is called a *false-play* if either Player I plays the  $\langle I've\ lost \rangle$  move in  $s$  or if  $s$  is a genuinely infinite play that contains an even number of negative literals.

We are now in a position to give a formal definition of the payoff function  $\Phi$ :

$$\Phi(s) = \begin{cases} T, & \text{if } s \text{ is a true-play} \\ F, & \text{if } s \text{ is a false-play} \\ 0, & \text{otherwise} \end{cases}$$

Notice that in the above definition of the payoff function, the value 0 corresponds to the case where  $s$  is a genuinely infinite play that contains an infinite number of negative literals i.e. there is an infinite number of role switches in the play.

**Corollary 1.** Let  $P$  be a program,  $G$  a goal clause and let  $\Gamma_{P_G}$  be the corresponding game. Then,  $\Gamma_{P_G}$  is determined.

Since the negation game is determined, we have the following definition:

**Definition 3.** Let  $P$  be a program. We define the game interpretation  $N_P$  of  $P$  as the interpretation such that for every  $p$  in the Herbrand base  $B_P$  of the program,  $N_P(p)$  is equal to the value of the game  $\Gamma_{P \cup \{\leftarrow p\}}$ .

The following theorem states that the game interpretation of a program is actually a model of the program:

**Theorem 1.** Let  $P$  be a program. Then,  $N_P$  is a model of  $P$ .

The above theorem provides a novel, purely game-theoretic characterization of the semantics of negation in logic programming. Subsequently, we investigate how this approach relates to the existing semantic approaches for negation and we prove the equivalence between  $N_P$  and the well-founded model semantics.



**Theorem 2.** *Let  $P$  be a program and let  $p$  be an atom that appears in  $P$ . Consider the goal  $G = \leftarrow p$  and let  $\Gamma_{P_G} = (X, T_\omega, \{F, 0, T\}, \Phi)$  be the corresponding (unrefined) game. Moreover, let  $M$  be the well-founded model of  $P$ . Then,  $\Gamma_{P_G}$  has value  $v \in \{F, 0, T\}$  if and only if  $M(p) = v$ .*

In order to prove the above theorem we use a new refined game that has infinite possible outcomes. The value of that game depends on the number of role switches that take place during it. The value of the game is  $T_k$  (respectively  $F_k$ ) if Player I (respectively Player II) wins after  $k$  role-switches.

**Corollary 2.** *Let  $P$  be a program,  $G$  a goal clause and let  $\Gamma_{P_G}$  be the corresponding refined (infinite-valued) negation game. Then,  $\Gamma_{P_G}$  is determined.*

**Theorem 3.** *Let  $P$  be a program and let  $p$  be an atom that appears in  $P$ . Consider the goal  $G = \leftarrow p$  and let  $\Gamma_{P_G} = (X, T_\omega, V, \Phi)$  be the corresponding refined (infinite-valued) game. Moreover, let  $M_P$  be the minimum infinite-valued model of  $P$ . Then,  $\Gamma_{P_G}$  has value  $v \in V$  if and only if  $M_P(p) = v$ .*

Therefore, the semantics captured using the refined game is equivalent to the minimum infinite-valued model of the logic program.

## Game Semantics of Intensional Logic Programming

We introduce a two-player game  $\Gamma_P(C, w)$  during which, Player I has the role of the *Doubter* and Player II the role of the *Believer*.

The infinite tree  $T_\omega$  of the game  $\Gamma_P(C, w)$  consists of all the infinite sequences  $\langle x_0, x_1, \dots, x_k, \dots \rangle$  which satisfy the following restrictions (in which  $A$  denotes a propositional atom,  $B_i$  denotes an intensional atom,  $u, v, z, y$  denote elements of  $W$  and  $S, S'$  denote subsets of  $W$ ) for each  $k \geq 0$ :

- R<sub>1</sub>**:  $x_0 = \langle C, w \rangle^-$ .
- R<sub>2</sub>**: If  $x_k = \langle \nabla A, u \rangle^-$ , then  $x_{k+1} = \langle A, S \rangle^+$ , where  $u \in \|\nabla\|(S)$ .
- R<sub>3</sub>'**: If  $x_k = \langle A, S \rangle^+$  and  $x_{k-1} = \langle \nabla A, u \rangle^-$ , then either **(i)**  $x_{k+1} = \langle A, v \rangle^-$ , where  $v \in S$ , or **(ii)**  $x_{k+1} = \langle A, S, S' \rangle^+$ , where  $S' \supset S$  and  $u \notin \|\nabla\|(S')$ . A move of type **(ii)** will be called a *role switch*.
- R<sub>3</sub>''**: If  $x_k = \langle A, S, S' \rangle^+$ , then  $x_{k+1} = \langle A, y \rangle^-$ , where  $y \in (S' - S)$ .
- R<sub>4</sub>**: If  $x_k = \langle A, v \rangle^-$ , then  $x_{k+1} = \langle \mathcal{C}, z \rangle^+$ , where  $\mathcal{C}$  is a clause in  $P$  of the form  $B_0 \leftarrow B_1, \dots, B_n$ , such that either **(i)**  $B_0 = A$  and  $z = v$ , or **(ii)**  $B_0 = \nabla A$  and for every  $S$  satisfying  $z \in \|\nabla\|(S)$  it holds  $v \in S$ .
- R<sub>5</sub>**: If  $x_k = \langle B_0 \leftarrow B_1, \dots, B_n, z \rangle^+$ , then  $x_{k+1} = \langle B_j, z \rangle^-$ , for some  $j$  with  $1 \leq j \leq n$ .
- R<sub>6</sub>**: If after  $x_k$  has been played, none of the above rules is applicable, then  $x_{k+1} = \langle I've\ lost \rangle$ .
- R<sub>7</sub>**: If  $x_k = \langle I've\ lost \rangle$ , then  $x_{k+1} = \langle I've\ won \rangle$  (and vice-versa).

Some explanations are in order. Suppose that  $C = \nabla A$  (the explanation for  $C = A$  is similar). Initially, Player I plays the move  $\langle \nabla A, w \rangle^-$ . The intuitive explanation for this move is “*I doubt that  $\nabla A$  is true in world  $w$* ”. Player II

believes the truth of  $\nabla A$  in world  $w$  and for this reason he replies to the move of Player I with a pair  $\langle A, S \rangle^+$ , where  $w \in \|\nabla\|(S)$ . The explanation for this move is “*I believe that  $\nabla A$  is true in  $w$ ; actually, I believe  $A$  is true in all the worlds contained in  $S$  and this implies that  $\nabla A$  is true in  $w$* ”. Player I now responds with a pair  $\langle A, v \rangle^-$  where  $v \in S$ . The intuition now is: “*I doubt that  $A$  is true in the world  $v$  of  $S$  (and therefore I continue to believe that  $\nabla A$  is not true in  $w$ )*”. Player II must now establish that  $A$  is true in  $v$ . One way to achieve this is to use a clause with head  $A$ . A second (less direct) way is to prove that  $\nabla A$  holds at some world  $z$  with the property mentioned in Case (ii) of rule **R<sub>4</sub>**; this property guarantees that if  $\nabla A$  holds at  $z$ , then  $A$  holds at  $v$ . Therefore, Player II provides a pair  $\langle C, z \rangle^+$  where  $C$  is a program clause with head  $A$  or  $\nabla A$ . If the head is  $A$  then  $z$  coincides with  $v$ ; otherwise  $z$  is selected so that for all  $S \subseteq W$  satisfying  $z \in \|\nabla\|(S)$  it holds  $v \in S$ . The intuition is “*Using this rule and the context  $z$  I can establish that  $A$  is true in the world  $v$* ”. Now Player I responds with a pair of the form  $\langle B_i, z \rangle^-$ , where  $B_i$  is one of the intensional atoms in the body of the rule that Player II has just played. The intuition is “*I doubt that  $B_i$  is true in world  $z$* ”.

As the game proceeds, the two players may swap roles (the Believer may become Doubter and vice-versa). Suppose now that at some point of the game, the Believer replies to a move of the form  $\langle \nabla A, u \rangle^-$  of the Doubter by playing  $\langle A, S \rangle^+$ , where  $u \in \|\nabla\|(S)$ . The Doubter can respond in two different ways. His first option is to play  $\langle A, v \rangle^-$  where  $v \in S$ . The intuition is: “*I doubt that  $A$  is true in the world  $v$  of  $S$  (and therefore I continue to believe that  $\nabla A$  is not true in  $u$ )*”. Alternatively, the Doubter can play  $\langle A, S, S' \rangle^+$  where  $S' \supset S$  and  $u \notin \|\nabla\|(S')$ . The intuition here is “*I believe that the set of worlds where  $A$  is true is  $S' \supset S$  and not  $S$  (as the Believer just claimed); since  $u \notin \|\nabla\|(S')$ , I was right in my belief that  $\nabla A$  is not true in  $u$* ”. This second type of move has made the player that was a Doubter to become a Believer and his opponent to become a Doubter (*role-switch*). In move **R<sub>3</sub>''**, the new Doubter plays  $\langle A, y \rangle^-$  where  $y \in (S' - S)$ . The intuition is “*I doubt that the set of worlds where  $A$  is true coincides with  $S'$ ; more specifically, I doubt that  $A$  is true in the world  $y$* ”.

The set of rewards is  $D = \{0, \frac{1}{2}, 1\}$  i.e., a play of the game can be assigned the value 0 (Player I has won the play), value 1 (Player II has won), or the value  $\frac{1}{2}$  (the result is a tie). Finally, the payoff function is defined as follows:

$$\Phi(s) = \begin{cases} 1, & \text{if Player II plays the } \langle I've \text{ won} \rangle \text{ move in } s \text{ or } s \text{ is a genuinely} \\ & \text{infinite play that contains an odd number of role-switches} \\ 0, & \text{if Player I plays the } \langle I've \text{ won} \rangle \text{ move in } s \text{ or } s \text{ is a genuinely} \\ & \text{infinite play that contains an even number of role-switches} \\ \frac{1}{2}, & \text{if } s \text{ contains an infinite number of role-switches} \end{cases}$$

According to the above definition, a player wins a play of the game if he manages either to play the  $\langle I've \text{ won} \rangle$  move or to remain the doubter after a certain point of the play; otherwise the result of the play is a tie.

**Theorem 4.** *Let  $P$  be a program,  $C$  be a propositional or intensional atom and  $w \in W$ . Then, the game  $\Gamma_P(C, w)$  is determined.*

**Definition 4.** Let  $P$  be an intensional logic program. We define the game interpretation  $N_P$  of  $P$  such that for every propositional atom  $A$  that appears in  $P$  and for every  $w \in W$ ,  $N_P(A)(w)$  is equal to the value of the game  $\Gamma_P(A, w)$ .

**Definition 5.** Let  $P$  be an intensional logic program and assume that the two-valued denotations of all intensional operators in the heads of the clauses of  $P$  are universal, monotonic and conjunctive while the two-valued denotations of the intensional operators in the bodies of clauses are arbitrary functions in  $\{0, 1\}^W \rightarrow \{0, 1\}^W$ . We define the game interpretation  $N_P$  of  $P$  to be the interpretation which for every propositional atom  $A$  that appears in  $P$  and for every  $w \in W$  has the property that  $N_P(A)(w) = v$  if and only if the value of the game  $\Gamma_P(A, w)$  is equal to  $v$ .

**Lemma 1.** Let  $P$  be a program and assume that the denotations of all intensional operators that appear in the heads of the clauses in  $P$  are universal, monotonic and conjunctive, while the denotations of intensional operators in the bodies of clauses are arbitrary functions in  $\{0, 1\}^W \rightarrow \{0, 1\}^W$ . Then, the game interpretation  $N_P$  of  $P$  is a model of  $P$ .

**Theorem 5.** Let  $P$  be a program and assume that the denotations of all intensional operators in the heads of the clauses of  $P$  are universal, monotonic and conjunctive while the denotations of the intensional operators in the bodies of clauses are arbitrary functions in  $\{0, 1\}^W \rightarrow \{0, 1\}^W$ . Then, the game interpretation  $N_P$  of  $P$  is a minimal model of  $P$  with respect to  $\preceq$ .

If we restrict our focus to monotonic programs, we have a simpler version of the game, with two values (0, 1) and without role switches. This game is also proved to be determined and it gives a game interpretation that is a model of the program and is identical to the unique minimum model  $M_P$  of [24]:

**Theorem 6.** Let  $P$  be an intensional logic program and assume that the denotations of all intensional operators in the heads of the clauses are universal, monotonic and conjunctive, and the denotations of all intensional operators that appear in the bodies of the clauses are monotonic. Then, the minimum intensional model  $M_P$  and the game interpretation  $N_P$  of  $P$  coincide.

### 3 Conclusions

In this work we presented infinite-game semantics for logic programs. Initially, we proposed an infinite-game characterization of the well-founded semantics for function-free logic programs with negation. The game is a simple generalization of the standard game for negation-less logic programs introduced in [34] in which two players, the *Believer* and the *Doubter*, compete by trying to prove (respectively disprove) a query. The game for programs with negation that we proposed follows the same rules as the standard one, except that the players swap roles every time the play "passes through" negation. We showed the *determinacy* of the new game by using some classical tools from the theory of infinite-games.

Our determinacy result immediately provides a novel and purely game-theoretic characterization of the semantics of negation in logic programming. More specifically it is equivalent to the well-founded semantics of logic programming.

In order to prove that equivalence, we defined a refined version of the game, i.e. an infinite-valued game that uses infinite degrees of winning and losing for the two players. We then demonstrated that this refined game corresponds exactly to the infinite-valued minimum model semantics of negation of [28]. This implied that the unrefined game is equivalent to the well-founded semantics.

The study continued with *Intensional logic programming*, an extension of logic programming, which includes as special cases both *temporal* and *modal* logic programming. A new game was defined and shown to be determined and equivalent to the semantics of M. Orgun and W. W. Wadge [24] for the case of programs in which the denotations of intensional operators in the heads of the clauses are monotonic, universal and conjunctive and the denotations of intensional operators in the bodies of the clauses are monotonic.

We then extended the game to a three-valued one that also applies to programs with non-monotonic operators and showed its determinacy. We proved that this extended game provides minimal model semantics for intensional logic programs. This way we have introduced the first (to our knowledge) general semantic framework for non-monotonic intensional logic programming. The proposed game can be used as a yardstick in order to develop alternative semantical approaches for non-monotonic temporal and modal languages.

There are many aspects of this work that we feel that should be further investigated. First of all, the (unrefined) negation game could apply as it is to infinite propositional programs. However, the proof of correctness has to be more involved. This is mainly due to the fact that the construction of the well-founded model of an infinite propositional program may require a transfinite number of iterations. This is also reflected in the construction of the minimum infinite-valued model of such programs: the set  $V$  of truth values contains a  $F_\alpha$  and a  $T_\alpha$  for each countable ordinal  $\alpha$  (see [28] for details). Therefore, in the correctness proof for the case of infinite programs, one has to appropriately redefine the refined game so as that the payoff function ranges over this new extended set of truth values. In the theory of infinite games such a situation is usually treated by introducing an auxiliary ordinal in the game that can be considered as a type of clock which imposes a “time limit” to the moves of the players (see for example [31]). A first attempt towards this direction appears in [11].

A game semantics for (negation-free) disjunctive logic programming, similar to our approach, has recently been developed in [32]. It would be interesting to further broaden our understanding regarding the interplay between logic programming and game-theory, by extending the game semantics to apply to other logic programming languages since many recent results ([6, 4, 10, 9, 11, 32]) and the present work suggest that this is a fruitful avenue of research. For example, it would be desirable to devise a game semantics for answer-set programming.

The use of any new semantic approach for a programming language, can only be tested by its applications. It would therefore be interesting to apply

the proposed approach in order to establish properties of logic programs that use well-founded negation. We conjecture that the game semantics can be used to demonstrate the correctness of program transformations as well as to define new ones. Since games are intuitive and natural, it is interesting to investigate whether they can offer certain benefits when compared against the classical semantics approaches.

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# Dynamically Adaptive Networks and provision of flexible services: Dynamically self-organized communities for content distribution and resource sharing

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**Abstract.** The work described in this thesis targets the introduction of an architecture in order to combine the two alternative models, and also exploit self-organization features of future networks between nodes of close proximity. The participating nodes form loosely coupled communities that enhance node cooperation for increased content distribution efficiency. Each node first searches for content to his peer members within the community, and if this cannot be satisfied the node resorts to the use of the infrastructure network. The provisioning of local content on each node is reciprocated with additional external bandwidth by the network gateway. The node cooperation is enforced by specific incentives that facilitate the node collaboration and deterrent measures that inhibit misbehaving nodes. In this work, the introduced architecture considers initially a centralized approach, then a distributed approach, and finally it is mapped to a SDN and mobile cloud enabled solution. The decision making mechanisms considered for the self-organization of the nodes is based on a theoretical model using game theory. Finally, simulation results for the proposed approach are presented to demonstrate the performance gains.

## 1 Introduction

In today's networks the content distribution landscape is changing. The evolution of mobile end user devices, with increasing processing, memory, storage, and display capabilities is changing the approaches for content placement[4], [5], [6]. Content is not restraint only to centralized storage servers but is also migrated towards end user devices, where it is transported and shared[1]. Furthermore, we experience a proliferation in user generated content, driven again by the evolution of end user devices. There is a diffusion among content producers, distributors and consumers. Users can operate on any of these roles. For the distribution of the increasing content volume, the two main approaches are the use of peer

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to peer networks and social networks. Peer to peer content distribution is used for high volume content transfers, usually in the case of high quality content. Besides the traditional bulk transfer mode, peer-to-peer streaming services are also becoming popular to provide on demand content to the members. On the other hand, social networks have become perhaps the most prominent means of content distribution. People use social networks, not only to keep in touch with friends and acquaintances but also to stay informed on news and events. Furthermore, people form groups of similar interests to exchange information and ideas. Most of today's web traffic is created by the use of social networks. This has a big impact on the network to provide the communication infrastructure and respective services for supporting the increasing volume of traffic for content distribution.

Both of the two predominant Internet application architectures (i.e., client-server and peer-to-peer (p2p)) essentially assume that nodes operate in isolation, even when close proximity exists. There are many cases where one user wants to retrieve or share information or content (even in real time) with other nearby users, but the communication has to take place over centralized services through the Internet. The work described in this thesis targets the introduction of an architecture in order to combine the two alternative models, and also exploit self-organization features of future networks between nodes of close proximity. The participating nodes form loosely coupled communities that enhance node cooperation for increased content distribution efficiency. Each node first searches for content to his peer members within the community, and if this cannot be satisfied the node resorts to the use of the infrastructure network. The provisioning of local content on each node is reciprocated with additional external bandwidth by the network gateway. The node cooperation is enforced by specific incentives that facilitate the node collaboration and deterrent measures that inhibit misbehaving nodes. In this work, the introduced architecture considers initially a centralized approach, then a distributed approach, and finally it is mapped to a SDN and mobile cloud enabled solution. The decision making mechanisms considered for the self organization of the nodes is based on a theoretical model using game theory. Finally, simulation results for the proposed approach are presented to demonstrate the performance gains.

The main outcomes of the work is in future networks we expect to have big gains in overall network resource usage by reducing the traffic to the main network infrastructure when exploiting the proximity and collaboration of nodes. Moreover, there is also gain on the node side in terms of delay in content acquisition.

## **2 Dissertation summary**

This dissertation presents in details the proposed architecture, and the network protocol to implement. The architecture is originally developed using a centralized approach, then a distributed approach and finally a solution based on Software Defined Networking (SDN) and mobile cloud. The internal architecture of



communities follows a flat structure using two main entities: participating nodes and the leader of the community. Furthermore, for the mechanisms implementation the assistance of network gateway and the mobile cloud infrastructure, in latest case, are required. The nodes participate in the content distribution, either as providers or consumers or intermediaries, vote in the voting process for other nodes that cooperated, report misbehaving nodes and finally can claim better service quality from the network gateway if they have acquired sufficient number of votes. Furthermore they offer a piece of their memory in order to create a global memory of the community, which may be used by other nodes as temporary storage. The leader of the community manages the community, collects votes, allocates additional bandwidth to nodes and advertises the community in order to enroll new nodes. The network gateway manages the bandwidth used to connect to the Internet and distributes it to the nodes according to the instructions of the community leaders. This is achieved by reserving a percentage of the total outgoing bandwidth and has offers it to the community leaders to distribute it to their members.

In the centralized approach the role of the community leader is undertaken by the network gateway. The network gateway is responsible for the construction, community operation and the bandwidth sharing. The network gateway initially builds the community by asking local nodes to the tags and size of content they already have or tags that interests them. Then it starts advertising the community to new nodes entering the region. The advertisement includes the community identifier, community tags. Furthermore it includes the size of the content size for each tag and the memory that the new node must offer to enter the community. After the mutual agreement the new node enters the community, and he participates in the content distribution, where for each file that he retrieves through the community he returns a proof the provider node. This proof is the vote used during the voting process, as presented and the corresponding vote counting algorithm. Finally, the reporting mechanism and additional functions of community as the content advertisement of the nodes are described. For the theoretical study of cooperation incentives, game theory was used. The nodes throughout this thesis are considered to behave selfishly, but rationally. Each node attempts to maximize his profit, but will not take an action that would be against his interest. Two games are described: the admission of a new member in the community, and the nodes' cooperation within the community. In the new node admission is shown that through the advertisement values of the community and the corresponding supply of the node, the cooperation is the most effective strategy only if and both sides believe that they will benefit. In this way community leader makes an initial filtering to the community members without allowing the admission of nodes who do not offer, while the nodes do not participate in a community that they offer too much. In the cooperation game it is proven that as the nodes try have better quality of service to the Internet, the best strategy is to collaborate with the other members.

The main drawback of the centralized approach are the number of changes that have to be made on the network gateway. Moreover, the functioning of

communities is not as autonomous, since an external entity is participating in the internal community operations. Furthermore, from the description the centralized approach, there is one community that strives to meet the needs of the majority of the nodes in the region. It would be easy to extend this so that the network gateway supports more than one community, but this would lead to an increase of the complexity on the gateway itself. Therefore the distributed approach is proposed[3]. The role of the community leader is undertaken by a community member. In this way more than one community can coexist in an area and a node can participate in more than one. The network gateway distributes the reserved bandwidth to the communities depending on the size of each community and guarantees that nodes have the additional bandwidth that they have gained. In the distributed approach, because the leader community is a node with limited resources, it is necessary to create safeguards in cases where there is either voluntary withdrawal or failure of this node. Finally, the protocol is extended to include communication of community leader with the network gateway for the bandwidth allocation. The assumption that the community in centralized approach is fair (the portal has no profit by acting unfairly) does not apply in the distributed approach, as the leader may take advantage of the extra bandwidth for his own exclusive use. This phenomenon is studied using game theory. In the distributed approach, the node model is extended and a separation of the nodes is introduced in nodes looking for more bandwidth compared to nodes seeking more content. The game between the community leader and participating nodes participants shows that if the leader it is not fair to the members, they cooperation among the members breaks, members leave the community and the community shrinks till it stops existing.

The distributed approach requires from the community leader to consume resources for its operation, forcing consequently only a certain class of nodes to play this role and for small time periods. To solve this issue an approach that is based on the use of SDN mobile cloud is used[2]. In this approach, the role of leader remains in the members of community, but moreover resources are committed centrally from the cloud infrastructure, which they undertake the demanding community operation. The auxiliary resources undertake the community's control plane, while the management plane and final decisions remain in the community leader. There are essential changes in the protocol for the offloading of operations in the auxiliary infrastructure. Based on this approach, the study of the community advertised values is presented and it is proved that for smooth and fair operation of community the optimal solution is the use of means values within the community. Finally, the behavior of the leader regarding the advertised prices is studied. The leader can select to advertise bigger or smaller values than the average. In any case via the game presented that is played between the community leader and the members of his community, it is proved that the advertisement of false values leads to the shrinkage of community.

### 3 Results

The evaluation of the architecture is based on two axes: the viability of the communities, and a analysis of optimization that can be achieved. For the viability two simulations were executed. The first simulation studied the size of the community, of the community content and the global memory. For the modeling the games described in the architecture presentation were used. The results shown in figure 1 show that the size of community increases, while at the same time the total content and the global memory also increase linearly with size of community. Consequently, using this architecture homogeneous communities are created, in which there are no nodes that offer in excess or nodes that don't offer back. The second simulation for the viability shows what happens in the case that a community leader is not fair to the rest of the members. Two cases were simulated. In the first case two communities existed with fair leaders. In second case in one community the leader was fair, while in the other unfair. As it is shown in figures 2 and 3 in the first case the communities continued keeping or slightly increasing the number of their nodes. On the contrary, in the second case the community with the unfair leader a mass departure of nodes took place after some time resulting to the extinction of the community. This simulation strengthens the results that were presented in the game theoretic study.

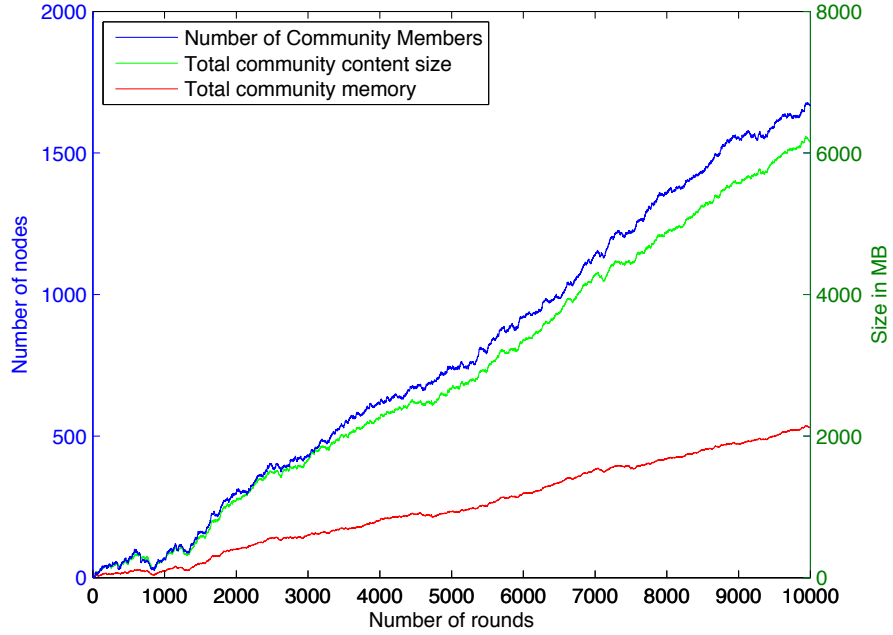
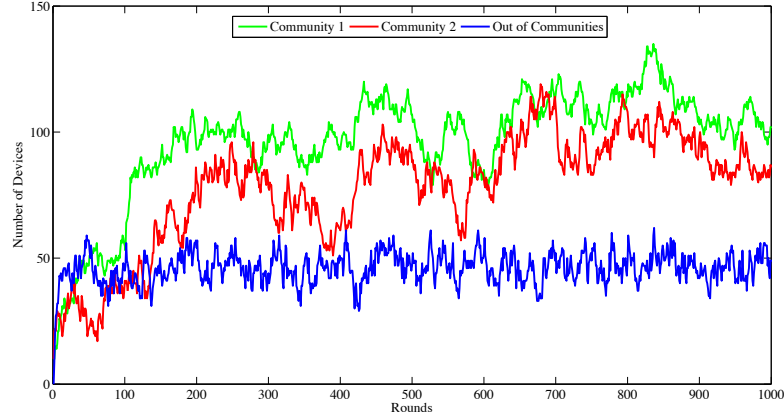


Fig. 1. Number of nodes per round

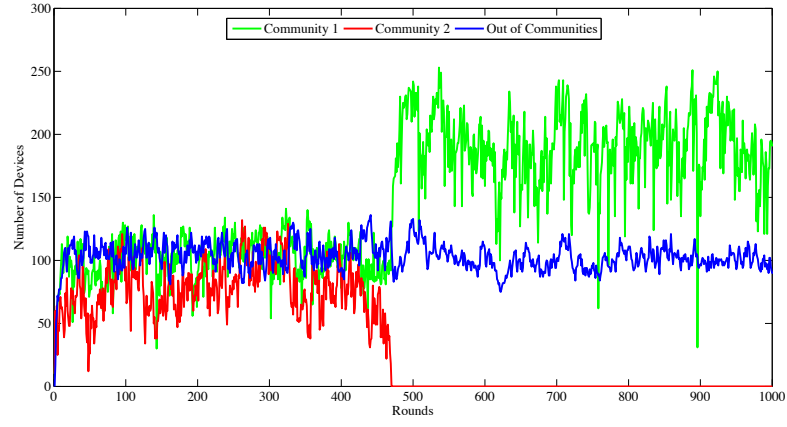


**Fig. 2.** Number of nodes in communities with fair leaders

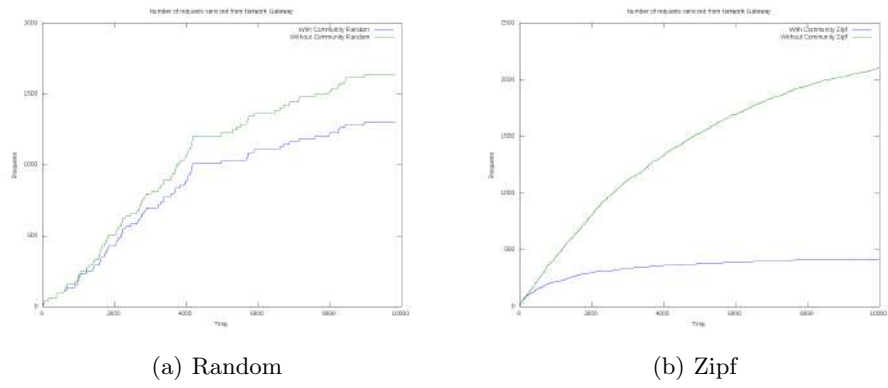
For the evaluation of the optimization that is achieved with this architecture two simulations were performed. The first analyzed the reduction of the external network load due to the content distribution inside the community. We simulated a number of nodes that made constant requests to the network gateway to acquire content. Without the use of the architecture all the requests have to be serviced by the network gateway, while with the use of the community a number of requests were served by the nodes. The results shown in figure 4 show a reduction of external requests ranging from 20 %, if the file choice was made using a uniform distribution, up to 400 %, if the file choice was made using a zipf distribution. In the second simulation we studied the optimization in content distribution inside the community. We compared per time needed to download a number of files using the Bittorrent protocol with and without the existence of the community architecture. The number of community members, as well as the mobility range of the nodes was varied. The results depicted in figure 5 show that in small mobility range of nodes the use of community has marginal benefits, but as long as the range grows then the benefits increase dramatically reaching improvement of 90 %. The results show that there are concrete benefits also in the end user content distribution.

## 4 Conclusions and future work

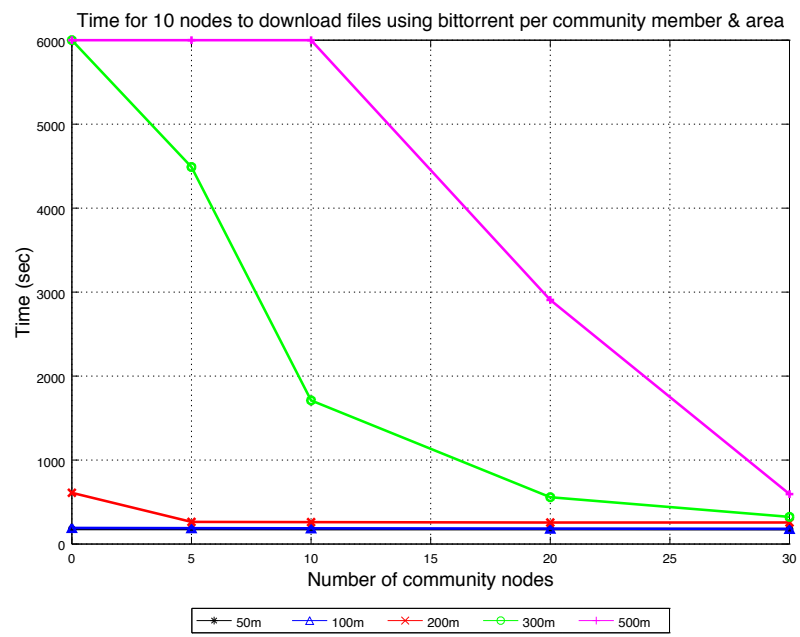
The overall conclusions of the thesis are that the presented architecture can offer benefits in the network infrastructure by reducing the load on the external connections. Moreover there are benefits on the user side by offering better content distribution. These benefits stem from the cooperation of nodes that are in close proximity. Future extensions of this work can be made in various axes. The first axis is the thorough study of user behavior within the communities in order to



**Fig. 3.** Number of nodes in communities with fair and unfair leader



**Fig. 4.** Number of requests through gateway with and without the community (20 nodes)



**Fig. 5.** Aggregation of time to complete download of files per number of community members and area (10 downloading nodes, 9 files)

maximize their gains by selecting content that can be exchanged for better network service. Furthermore, the content replacement policies when user devices fill up, should be augmented to accommodate not only the frequency of use by the user, but also the value of the content within the community. The second axis is the study of closed coupled communities that can offer better orchestration of operations. In this case mechanisms should be used that guarantee stricter commitment from the members, making possible the better orchestration of content distribution. Finally, the third axis is the generalization of architecture beyond content distribution and extend it in order to make possible more resources like CPU cycles. In this case a need for a digital currency (like Bitcoin) is required to provide the common ground for the resource exchange to take place.

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# Nanophotonic Slow-Light Structure For Telecommunication Applications

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**Abstract.** In this thesis, the delay performance of slow light optical pulses inside PCWs is considered in the linear and nonlinear propagation regime from both a theoretical and an application point of view. It is numerically shown that for rates of  $40Gb/s$  and  $100Gb/s$ , nonlinear solitary pulses experience less broadening than the linear case and can therefore be used to obtain larger delays. The storage capacity of slow light PCWs is maximized using a systematic procedure based on the optimization of various parameters of the structure. Moreover, approximate analytical expressions for the estimation of the degenerate four-wave mixing (FWM) conversion efficiency in slow-light PCWs are presented. The derived formulas incorporate the different effective modal areas and the frequency-dependent linear and nonlinear parameters of the pump, signal, and idler waves. The influence of linear loss, two-photon absorption, and free-carrier generation is also accounted for. We discuss the optimization of PCWs for FWM applications, taking into account linear loss and free-carrier effects. Suitable figures of merit are introduced in order to guide us through the choice of practical, high-efficiency designs requiring relatively low pump power and small waveguide length. Promising waveguide designs are identified, altering some structural parameters. These designs are identified using an optimization process taking into account sophisticated figure-of merits that depend on the pump bandwidth and the signal/pump tunability. We also present alternative designs that are less efficient but have smaller power requirements and are far more compact.

**Keywords:** Photonic crystal waveguides, Slow-light, Four-wave mixing, Soliton, Delay Lines.

## 1 Introduction

Photonic crystals are formed by periodically modulating the refractive index of the material in all three directions. Such structures are known to prevent light from propagating in certain directions with specified frequencies, an ability usually referred to as photonic band gap. Researchers devote a considerable amount

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of attention to photonic band gaps and with good reason. Many of the promising applications of two- and three- dimensional photonic crystals to date hinge on the location and width of photonic band gaps. For example, a crystal with a band gap might make a very good, narrow-band filter, by rejecting all frequencies in the gap. A resonant cavity, carved out of a photonic crystal, would have perfectly reflecting walls for frequencies in the gap. The simplest possible photonic crystal consists of alternating layers of material with different dielectric constants. A two-dimensional photonic crystal is periodic along two of its axes and homogeneous along the third axis. Usually a two dimensional photonic crystal is formed by embedding holes of a low refractive index material in a triangular lattice, to a higher refractive index material. Another way is to form a square lattice of dielectric columns in a lower dielectric environment. The latter is a less attractive option because such photonic crystals experience a narrower band gap as well as increased linear losses due to light scattering. In a practical application, light must be confined in all three dimensions, necessitating the usage of a three dimensional photonic crystal, which is a dielectric structure with periodicity along three different axes. However, a three dimensional photonic crystal has certain weaknesses in both fabrication and practical application. In practice it is more common to combine band gap with index guiding, creating photonic crystal slabs. A photonic crystal slab is a hybrid structure formed by adopting a two dimensional photonic crystal structure and confine light in the third dimension through means of internal reflection. For example, as a photonic crystal slab can be considered a silicon membrane embedded with holes of air in a triangular lattice. In this case, light will be confined in the third direction by layers of a lower dielectric material above and below the slab. In case these layers are filled with air, the photonic crystal is called air membrane PCW.

Introducing a defect in the photonic crystal, (i.e. by removing a line of holes along the propagation direction of a photonic crystal slab), a defect mode (or guided mode) appears inside the photonic band-gap. The localization of the waveguide mode relies on both the band gap within the plane of periodicity and also on index guiding in the vertical direction. One of the remarkable properties of this mode is that at a given frequency range, propagation occurs with an increased group index  $n_g = c/|v_g|$ , where  $v_g$  is the group velocity (which may be positive or negative depending on the slope of the dispersion curve) and  $c$  is the speed of light in vacuum. This phenomenon is widely known as slow-light.

## 2 Slow-Light

In this dissertation an extended description of the slow-light effect is presented. In general, slow light occurs due to large first order dispersion  $dk/d\omega$  arising from the resonance of light with a material or structure, where  $k$  is the wave number and  $\omega$  is the angular frequency. The most noticeable method that uses material dispersion in order to manipulate light is the electromagnetically induced transparency (EIT). This method holds the slow-light record at 17m/s

using Bose-Einstein Condensates (BEC). The extremely low  $v_g$  in EIT only allows a bandwidth of the order of kHz. For structural dispersion methods,  $\Delta n$  is defined not for a material index but for an equivalent index of mode distributing over multiple materials that form the structure. Therefore, structural dispersion methods have similar problems in terms of the bandwidth and dispersion, although they are suitable for room temperature on-chip applications. Photonic crystals falls into the category of structural dispersion, forming standing waves on the Bragg condition of their periodic structure (usually called band edge) and slow light occurs due to large first order dispersion near the Bragg condition. Presently, it is straightforward to observe experimentally a group velocity of  $c/10$   $c/100$  and a delay of 10ps order.

### 3 PlaneWave Expansion Mode Solver

This thesis reviews the most important numerical techniques for the solution of partial differential equations that can be applied to obtain the band diagram, transmission spectra and field patterns of the photonic crystal slab waveguide (PCSW). Each numerical method has each own particular strengths and weaknesses. As thoroughly explained in this thesis, these numerical methods are solving the eigenvalue problem in time or frequency domain. In our calculations, we have implemented a three dimensional plane wave expansion method mode solver based on the minimization of the Rayleigh quotient (using the Rayleigh-Ritz method), which we have implemented in MATLAB. On a computer, this eigen-equation must be discretized into  $N$  degrees of freedom using the planewave expansion method. In general, such a discretization yields a finite generalized eigen-problem  $\mathbf{A}x = \omega^2 \mathbf{B}x$ , where  $\mathbf{A}$  and  $\mathbf{B}$  are  $N$  matrices and  $x$  is the eigenvector. Since the original eigen-problem is Hermitian, the discretization can be chosen so that  $\mathbf{A}$  and  $\mathbf{B}$  are Hermitian and  $\mathbf{B}$  is positive-definite. This realization leads to iterative methods, which compute a small number  $p$  of the eigenvalues and eigenvectors, such as the  $p$  smallest eigenvalues. There are many such methods, but they share a few critical features. Firstly, they work by taking a starting guess for  $x$  (e.g., random numbers) and applying some process to iteratively improve the guess, converging quickly to the true eigenvector. In this way, any desired accuracy can be obtained in a small number of steps. Secondly, they merely require you to supply a fast way to compute the matrix-vector products  $\mathbf{A}x$  and  $\mathbf{B}x$ . The dimensions of the supercell must be chosen carefully for two main reasons. Firstly, must be large enough to contain the photonic crystal defect that forms the waveguide. In addition, the supercell size must prevent coupling with unwanted waveguides, formed by the periodic repositioning of the supercell. Moreover, near a dielectric interface one must average the dielectric in two different ways according to effective-medium theory, depending upon the polarization of the incident light relative to the surface normal  $\hat{n}$ . As stated in this thesis, not doing so can lead to suboptimal convergence of the frequencies as a function of  $N$ , due to the problems of representing discontinuities in a Fourier basis. It has been shown, that using a smoothed, effective dielectric tensor near

dielectric interfaces can circumvent these problems, and achieve accurate results for moderate  $N$ . The desired modes (guided modes) of the photonic crystal waveguide lie in a known frequency range (the band gap) in the interior of the spectrum. Ideally, one would like to compute only the defect modes in the band gap, without waste computation and memory on finding all the folded modes below them. In this thesis, a technique for calculating only the guided mode located inside the photonic band gap is unfolded. Applying this technique one will see that despite the fact that much fewer eigenvalues need to be found, the convergence is slower than before due to lack of appropriate preconditioning. The fact that a more suitable preconditioner needs to be constructed makes the interior eigenvalue method the less efficient choice.

## 4 Major Limitations of Photonic Crystals

This thesis contributes to the debate and understanding of the propagation factors limiting the applicability of photonic crystal waveguides. Slow-light in photonic crystals tends to coincide with high dispersion, which removes most of the advantages of operating in the slow light regime and severely limits the bandwidth that can be utilized. Moreover, linear losses are another issue currently being debated. It has been proposed that losses in photonic crystal waveguides scale as the square of the slowdown factor,  $S = c/v_g$ . The dispersion effects as well as the linear loss level is modelled and examined for PCSW. It is shown that dispersion effects are not an intrinsic property of the structure, but subject to design. Designs based on a better understanding of slow light operation can overcome this limitation, as already shown by several authors. Propagation in the aforementioned slow light regime can find use in a variety of practical applications including optical delay lines and enhanced lightmatter interaction. Given the dispersion relation of the mode, the coefficients  $\beta_2$  and  $\beta_3$  can be easily extracted using polynomial fitting on the dispersion relation. The group velocity dispersion (GVD) coefficient,  $\beta_2$  is usually larger in the slow light regime, potentially leading to significant pulse broadening, especially at high data rates. To some extent, careful waveguide design can be applied in order to reduce the propagation losses while at the same time obtain lower values of  $\beta_2$ . The amount of pulse broadening can be quantified in terms of the broadening factor BF, which is defined by the ratio,

$$BF = \sigma(L)/\sigma(0) \quad (1)$$

,where  $\sigma(x)$  is the root-mean-square (RMS) pulsewidth. This thesis presents a BF study for several photonic crystal waveguide designs. The presented results suggests that GVD can cause a severe amount of broadening increasing for higher bit rates and that TOD induced broadening is significantly less important. This provides a first motivation for considering soliton pulses since, SPM can compensate for GVD-induced broadening. The soliton pulse shape depends on the sign of the GVD coefficient  $\beta_2$ . If  $\beta_2$  is positive (which is the case in the majority of the photonic crystal waveguides considered in this thesis), then dark solitons should be launched in the structure. The input soliton pulse-width  $T_0$  is

related to the corresponding full-width at half maximum duration,  $T_{FWHM}$ , of the pulse through  $T_0 = T_{FWHM} < 1.76$ . The pulse peak power  $P_0$  is determined by the following expression,  $P_0 = |\beta_2|/\gamma T_0^2$ . The presented results suggest that for the W1 waveguide,  $P_0$  is initially increasing with  $n_g$  and then saturates near  $n_g = 15$ . For  $n_g > 15$ ,  $P_0$  slowly decreases. This behavior is a consequence of the interplay between  $\beta_2$  and  $\gamma$ , which in the case of the W1 waveguide are both increasing monotonically with  $n_g$ . In the nonlinear regime, the residual pulse broadening due to the propagation losses and TOD can be studied by numerically solving the propagation equation using the split step Fourier (SSF) method. In order to simulate the propagation of dark solitons, measures should be taken to prevent the truncation of the bright background of the pulse at the edges of the SSF time window. The broadening factor can be calculated after removing the white background from the output waveform. The presented results show that for data rates of interest in optical networking (such as 40 Gb/s and 100 Gb/s), soliton pulses can be used to obtain significant improvement in terms of the achievable delay and broadening level compared to linear pulses. We also provide a comparison between the broadening factors obtained in the linear and the nonlinear regime for data rates of 40Gb/s and 100Gb/s. It is shown that for all waveguide designs, nonlinear propagation can be quite beneficial, especially at 100Gb/s, increasing the delay obtained from each particular waveguide at a given broadening factor. We also discuss the effect of varying loss level and the benefit of launching solitons at higher peak power. Chapter 4 also highlights the relation between the propagation losses, the achievable delay, and the amount of pulse broadening experienced in a photonic crystal slab waveguide in both the linear and the nonlinear regime. The propagation loss coefficient  $\Gamma$  is given by the expression,

$$\Gamma = c_1 \rho_{OP} n_g + c_2 \rho_{BS} n_g^2 \quad (2)$$

This expression corresponds to the total propagation losses that encompass the intrinsic loss, the disorder-induced scattering and the losses due to out-of-plane propagations losses. The coefficients  $c_1$  and  $c_2$  can be extracted from measurements and depend on the fabrication method, the dielectric contrast  $\Delta\epsilon$ , and the disorder parameter  $\sigma_d$ . The scattering coefficients  $\rho_{BS}$  and  $\rho_{OP}$  encompass the influence of the mode shape. They correspond to backscattering and out-of-plane scattering, respectively. In the waveguide designs under consideration, we have found that out-of-plane scattering has only marginal influence and hence can be ignored. Assuming that  $\rho_{OP} \cong 0$ , it is easy to relate the losses of any PCSW to those of a standard W1 waveguide fabricated with the same index contrast and disorder parameter, i.e.,

$$\Gamma(n_g) = \Gamma'(n_{g0}) \left( \frac{n_g}{n_{g0}} \right)^2 \frac{\rho_{BS}(n_g)}{\rho_{BS}(n_{g0})} \quad (3)$$

where  $\Gamma(n_{g0})$  is the loss coefficient of the W1 waveguide calculated at  $n_g = n_{g0}$  and  $\rho'_{BS}(n_{g0})$  is the corresponding backscattering coefficient of the W1 waveguide. This dissertation contributes also to the impact of multiple scattering in the propagation of the pulse. Multiple scattering is the process by which the

backscattered light is coupled back to the forward propagating mode and manages to reach the output. Multiple scattering manifests as a number of random peaks in the normalized transmission spectra of the waveguide. The measured linewidth of the fluctuations of the transmission of a photonic crystal waveguide is estimated to be about 10 GHz for  $\lambda = 1550nm$ . To illustrate the impact of these fluctuations, we consider the spectrum of a Gaussian pulse filtered by a randomly fluctuating transmission curve,

$$H(f) = \sum H_m e^{j2\pi m f / \Delta f}, \quad (4)$$

where  $\Delta f$  is the spectrum of the pulse and the index  $m$  runs as  $1mN_p$ .  $H(f)$  corresponds to the transmission curve of a waveguide segment comparable to the localization length. We choose  $N_p = 16$ , large enough in order to produce 10-GHz-spaced fluctuations. From the presented results it is deduced that the main pulse shape remains practically unchanged but a small part of the initial energy gives rise to nearby trailing small amplitude pulses (as expected in the case of a multiply backscattered signal). However, the pulse peak power and broadening factor remain practically unchanged, suggesting that propagation is not severely affected by this effect. The above considerations suggest that these narrow transmission fluctuations are not expected to severely impact the propagation of high bandwidth signals, which to a first approximation propagates as if these fluctuations are smoothed out.

## 5 Photonic Crystals and Four-Wave Mixing Phenomenon

This thesis contributes also to the four wave mixing phenomenon in photonic crystal waveguides. Four-wave mixing (FWM) is an important nonlinear phenomenon that may hold the key for many signal-processing applications in future optical networks, including wavelength conversion, signal regeneration, phase inversion, optical switching, and optical de-multiplexing. Degenerate FWM occurs when part of the optical power of a signal wave can be transferred to an idler wave located at another frequency through the mediation of a strong pump wave located at a third frequency. The efficiency of the energy exchange in this process is larger when the phases of the three waves are matched, i.e., when  $\phi = 2\phi_p\phi_s\phi_i$  is small, where  $\phi_p$ ,  $\phi_s$ ,  $\phi_i$  denote the total phase of the pump, signal, and idler waves, respectively. The most commonly adopted figure of merit that characterizes FWM is the conversion efficiency defined by,

$$\eta \equiv \frac{P_i(L)}{P_s(0)}, \quad (5)$$

where  $P_s(0)$  is the incident signal power, and  $P_i(L)$  is the idler power at the output of the waveguide of total length  $L$ . Nanophotonic slow-light structures such as photonic crystal waveguides offer the possibility of achieving sub-wavelength light confinement, while at the same time enhancing nonlinear effects such as

FWM. The estimation of  $\eta$  can play a crucial role in the design of the waveguide and guide us through the choice of several geometric, material, and signal parameters. In the case of degenerate FWM, the evolution of the three waves is generally described by a system of coupled ordinary differential equations. By solving this system of equations, one can in principle estimate  $\eta$ . Accounting for nonlinear losses complicates the problem, rendering the derivation of an exact analytical expression extremely difficult. In semiconductor materials such as silicon, nonlinear losses usually stem from two-photon absorption (TPA) and free-carrier (FC) generation. Self-phase modulation (SPM), cross-phase modulation (XPM), and dispersion should also be taken into account. Another complication arises from the fact that the wave parameters can exhibit substantial frequency dependence in PCWs, especially in the slow-light regime. Even if the waveguide is designed to ensure a smooth linear loss and group index frequency dependence, there is no guarantee that the nonlinear propagation parameters such as the effective modal areas  $A$  for all three waves will be the same. In fact, recent studies argue that SPM, XPM, and FWM may each perceive different values for  $A$ , unlike the case of a weakly guiding dielectric fiber, where such intricacies can be ignored. The results presented in this dissertation indicate that, the modal areas can exhibit strong frequency dependence even inside the flat-band region of the waveguide. It is therefore incorrect to assume the same modal area for all three waves, especially when the detuning is larger. Also the modal areas corresponding to each phenomenon may differ significantly in the case of large detuning. This thesis presents approximate analytical expressions for the FWM conversion efficiency  $\eta$ , when linear and nonlinear losses affect the propagation of the three waves. The usefulness of these formulas is twofold: first they provide significant insight into the nature of the FWM phenomenon from a theoretical point of view. They can also provide a target optimization function that requires much less computational time than the numerical solution of ODEs, when designing the PCW for nonlinear signal-processing applications. Unlike the design of PCWs for buffering applications, when optimizing the waveguide for FWM applications, one must also consider a multitude of signal parameters such as the pump-signal wavelength detuning and the incident pump power, which necessitate a large number of efficiency calculations for each structure. Simple analytical expressions can therefore speed up the optimization process. The approximate analytical expression of the FWM conversion efficiency  $\eta$ , assuming that only the nonlinear TPA loss is given by,

$$\eta = \left(\frac{\omega_i}{\omega_s}\right) \left(1 + \frac{\kappa^2}{4g^2}\right) \sinh^2(gL) e^{-\alpha_i L - 2ReT_i \overline{P_p} L}. \quad (6)$$

In the above equation,  $\omega_\mu = 2\pi c/\lambda_\mu$  where  $\lambda_\mu$  is the wavelength for wave  $\mu$ ,  $\kappa$  is the total phase mismatch and  $g$  is the parametric gain. The parameter  $T_i = (2jn_2\omega_i c^{-1} - \beta_{TPA})S_p S_i A_{pii}^{-1}$  where  $n_2$  is the nonlinear Kerr coefficient,  $\beta_{TPA}$  is the TPA coefficient,  $S_m$  is the slow-down factor of the  $m$  wave and  $A_{pii}$  is the effective modal area of the XPM. The average pump power is calculated

by using the derived expression,

$$\overline{P}_p = \frac{A_{ppp}}{\beta_{TPA} S_p^2 L} \ln \left( 1 + \frac{\beta_{TPA} S_p^2 P_p(0)}{\alpha_p A_{ppp}} [1 - e^{-\alpha_p L}] \right) \quad (7)$$

TPA is typically accompanied by FC generation causing an additional absorption and dispersion. As discussed by several authors, FC effects can be significantly reduced when either a low repetition/low duration pulsed pump is used or when an external DC field is applied driving the FCs away from the center of the waveguide. We have compared the efficiency values obtained by the presented expression against the numerical solution of the coupled ODE equations, with respect to the wavelength of the signal and the idler waves. In this thesis we visually infer that overall the approximate formula provides an adequate description for medium- to high-efficiency values, which are important from a practical point of view. To quantify the error in the approximation, we calculated the average error  $e_5$  and  $e_{10}$  between the numerical and the analytical efficiency (measured in dB) for wavelength combinations in which the ODEs efficiency is not lower than 5dB and 10dB compared to  $\eta_{max}$ , respectively. We obtain  $e_5 = 0.35dB$  and  $e_{10} = 1.1dB$ , implying very good agreement for efficiency values of practical interest.

If no measures are taken, the FC generation can severely limit the FWM conversion efficiency. When FC effects are included, deriving an analytical approximation for  $\eta$  is much more involved. For one thing, the pump power cannot be obtained in exact form as in the previous cases. To that end, this thesis present two alternative methods for obtaining  $P_p(z)$ , which can be used in the estimation of  $\eta$ . First we may assume that the three loss types (FC absorption, TPA, and linear loss) act independently and that the overall pump loss can be approximated by the product of the three loss factors. This assumption leads to the following expression for the pump power,

$$P_p(z) = P_p(0) \frac{e^{-a_p z}}{(1 + K_1 z) (1 + K_2 z)^{1/2}} \quad (8)$$

where we have defined the parameters  $K_1 = P_p(0) \beta_{TPA} S_p^2 / A_{ppp}$  and  $K_2 = 4P_p(0)^2 \text{Re}\{F_p\}$ . Adopting this first-order approximation for the exponential, we may readily obtain a closed-form formula for the average pump power:

$$\overline{P}_p = \left[ \frac{2(e_1 - e_0 K_1)}{L K_1^{3/2} \sqrt{K_1 - K_2}} \tanh^{-1} \left( \frac{\sqrt{K_1} \sqrt{1 + K_2 z}}{\sqrt{K_1 - K_2}} \right) + \frac{2e_1 \sqrt{1 + K_2 z}}{L K_1 K_2} \right]_0^L, \quad (9)$$

where  $[f(z)]_b^d = f(d) - f(b)$ . The coefficients  $e_0$  and  $e_1$  can be obtained so that the difference between the exponential and its first-order approximation is minimum in the least square sense inside  $[0, L]$ , in which case we find that,

$$e_0 = 2l_0^{-1} [(e^{-l_0} + 2) + 3l_0^{-1} (e^{-l_0} - 1)], \quad (10)$$

$$e_1 = -6l_0^{-1} L^{-1} [(e^{-l_0} + 1) + 2l_0^{-1} (e^{-l_0} - 1)], \quad (11)$$



with  $l_0 = a_p L$ . A second alternative is to solve the pump power in the case where the nonlinear loss is dominated by the FC absorption, i.e.,  $Re\{F_p\} P_p \gg Re\{T_p\}$ , in which case the pump power is given by the following equation,

$$P_p(z) = \frac{P_p(0)e^{-a_p z}}{(1 + \delta(1 - e^{-2a_p z}))^{1/2}}, \quad (12)$$

where  $\delta$  is given by  $\delta = 2a_p^{-1} Re\{F_p\} P_p(0)^2$ . Integrating with respect to  $z$  and dividing with the waveguide length, we readily obtain the following expressions for the average pump power and the average square pump power,

$$\bar{P}_p = -\frac{P_p(0)}{a_p L \sqrt{\delta}} \left\{ \sin^{-1} \left( \frac{e^{-a_p L}}{\sqrt{\delta^{-1} + 1}} \right) - \sin^{-1} \left( \frac{1}{\sqrt{\delta^{-1} + 1}} \right) \right\} \quad (13)$$

Once the pump power is obtained by either one of the two methods discussed above, we can proceed to the estimation of the approximate efficiency. We arrive at the expression for  $\eta_0$  given by,

$$\eta = l_i \left( \frac{\omega_i}{\omega_s} \right) \left( 1 + \frac{\kappa_{tot}^2}{4g^2} \right) \sinh^2(gL), \quad (14)$$

where the total phase mismatch  $\kappa$  is replaced by  $\kappa_{tot}$ , which is determined by,

$$\kappa_{tot} = \kappa + Im\{F_s + F_i - 2F_p\} \bar{P}_p^2. \quad (15)$$

We validate the results obtained by the above analytical formula considering both aforementioned methods for estimating the pump power. We compare the FWM conversion efficiency obtained analytically against the numerical solution with respect to the wavelength of the signal and the idler waves. The presented results show that an overall good agreement is obtained between the numerical and the analytical solution for medium- to high-efficiency values. The average error for  $(\lambda_i, \lambda_s)$  combinations for which the ODE efficiency is not smaller than 5 dB than  $\eta_{max}$  is  $\varepsilon_5 = 0.53dB$  and  $\varepsilon_5 = 0.28dB$  calculating the pump power with Eq. 8 and Eq. 12, respectively. The same case for which the ODE efficiency is not smaller than 10dB than  $\eta_{max}$  is  $\varepsilon_{10} = 2.03dB$  and  $\varepsilon_{10} = 1.95dB$ , respectively. The results obtained are calculated based on state-of-the-art fast-light linear loss levels and values of  $\beta_{TPA}$  corresponding to silicon. We note that the analytical formulas provide accurate results of the FWM conversion efficiency compared to numerical calculations. This thesis also briefly examines how the nonlinear loss due to FC generation and its impact on the efficiency  $\eta$  can be estimated in the case of a pulsed pump. The time evolution of the FC density,  $N_C$  is given by,

$$\frac{\partial N_C}{\partial t} = \frac{N_0 - N_C}{\tau_C} \quad (16)$$

, where  $N_0 = \beta_{TPA} \tau_C S_p^3 P_p^2(z, t) / 2\hbar\omega_p A_{ppp}^2$  is the FC density in the continuous-wave regime. We assume that the input pump signal has a period equal to  $T$

and is comprised of rectangular pulses and duration equal to  $T_1$ . We can assume that dispersion effects do not significantly affect the pulse shape. Therefore the pump pulse will approximately retain its rectangular shape along the propagation length, and only its peak power will decrease because of loss. Solving the above equation for the  $n^{th}$  pulse period  $[t_n, t_{n+1}]$  where  $t_n = nT$ , one finds that,

$$N_C(z, t) = N_C(z, t_n)e^{-(t-t_n)/\tau_C} + N_0 \left\{ e^{-t_n/\tau_C} - e^{-(t-t_n)/\tau_C} \right\} \quad (17)$$

during the on period of the pulse  $t_n < t_{n+1}$  and

$$N_C(z, t) = N_C(z, t_n + T_1)e^{-(t-t_n-T_1)/\tau_C}, \quad (18)$$

if  $t_n + T_1 < t_{n+1}$ . In the initial pulse periods (small  $n$ ), there will be a gradual buildup of FCs until one reaches a point where the FC density  $N_C(z, t_n)$  at the start of each period will be the same regardless of  $n$ . Under this condition, one obtains  $N_C(z, t_{n+1}) = N_C(z, t_n)$ , and combining the two last equations, we find that,

$$N_C(z, t_n) = N_0 \left\{ e^{-t_n/\tau_C} - e^{-T_1/\tau_C} \right\} \frac{e^{-(T-T_1)/\tau_C}}{1 - e^{-T/\tau_C}} \quad (19)$$

We can easily calculate the average carrier density  $N_{avg}$  inside the pulse duration and use this carrier density in the estimations of the loss coefficient. Our results indicate that as the repetition rate becomes smaller, at some point  $T$  becomes much larger than  $\tau_C$ , and the generated FCs have the necessary time to fully recombine before the next pulse arrives. Therefore in this regime,  $N_C(z, t_n) \simeq 0$ , and the nonlinear losses are due solely to the carriers generated inside the current pulse period, which do not depend on  $T$  and the repetition rate. As a consequence, the efficiency tends to remain constant at small repetition rates. For repetition rates above 1 GHz, an exponential degradation of  $\eta$  is observed. In this case, carriers generated in the previous pulse duration do not recombine fully, and there is a buildup of carriers, which increase the nonlinear loss.

## 6 Design Optimization

One of the main contributions of this thesis is the study of photonic crystal design with respect to the storage capacity. We define the storage capacity of the photonic crystal waveguides as the ratio of the achieved delay  $L_W/|v_g|$  to the bit duration  $1/R_b$ , i.e.,

$$N_{\max} = L_W R_b / |v_g|, \quad (20)$$

where  $L_W$  is the waveguide length and  $v_g$  is the group velocity of the defect mode that carries the signal.  $N_{\max}$  is therefore a function of  $R_b$  but also of the wave vector  $k$  and the geometry of the waveguide superlattice. Formally we can write  $N_{\max} = f(R_b, \alpha, k, r_\alpha, \varepsilon_\alpha, \varepsilon_b, h, x_1, y_1, r_1, x_N, y_N, r_N)$ , where  $N$  is the number of hole classes considered in the optimization,  $r_\alpha$  is the radii of the lattice holes,  $\alpha$  is the lattice constant, and  $\varepsilon_\alpha$  and  $\varepsilon_b$  are the relative dielectric constants of

the high- and low-index material, respectively. The function  $f$  is not known in closed form, but it can be computed using a plane wave expansion eigenmode solver to obtain the modal fields and the dispersion relation  $k = k(\omega)$  of the waveguide. We apply standard optimization methods to choose the arguments of  $f$  in order to maximize  $N_{max}$ . To estimate  $N_{max}$ , one must estimate  $L_W$ , which is determined by the maximum tolerable optical loss and the dispersion-induced pulse broadening. In this thesis, we consider that the loss limit is  $l_{max} = 20dB$ , which can be easily compensated by semiconductor optical amplifiers. Given the optical loss coefficient  $\Gamma$  of the waveguide in dB/cm, the maximum propagation distance due to losses is simply  $L_\Gamma = l_{max}/\Gamma$ . We also considered the maximum allowable length due to dispersion  $L_B$  is,

$$L_B = K(B_{max}^2 - 1)^{\frac{1}{2}} \left( \beta_2^2 R_b^4 + \frac{1}{4} K^{-1} \beta_3^2 R_b^6 \right)^{-\frac{1}{2}} \quad (21)$$

where,  $\beta_2$  and  $\beta_3$  are the GVD and third-order dispersion coefficients respectively,  $B_{max} = 1.3$  is the maximum allowable broadening factor, and  $K = 0.0224$ . We choose the maximum propagation length  $L_W$  as the minimum of  $L_B$ ,  $L_\Gamma$ , and  $L_{max}$  where  $L_{max} = 1cm$  is the waveguide length limit imposed by optical integration considerations. In this thesis, we also discuss how the optimization procedure can be applied in order to design a PCSW delay line from scratch, considering the effect of multiple design parameters. We choose the standard W1 waveguide as a starting point with  $r_\alpha = 0.27\alpha$ , and perform a step-by-step optimization gradually increasing the number of parameters considered. Our presented results revealed a design with optimum storage capacity  $N_{max} = 31.3bits$  at  $n_g \simeq 24$  for  $\Delta y_1 = 0.1297\alpha$ ,  $\Delta y_2 = 0.0248\alpha$ ,  $\Delta y_3 = 0.0399\alpha$ , and  $\Delta r_1 = 0.25\alpha$ , considering  $R_b = 40Gb/s$ . In the same chapter, we also examine the case of  $R_b = 100Gb/s$ , where the maximum capacity obtained was  $N_{max} \simeq 65bits$  at  $n_g \simeq 21$  for  $\Delta y_1 = 0.14\alpha$ ,  $\Delta y_2 = 0.025\alpha$ ,  $\Delta y_3 = 0.018\alpha$ ,  $\Delta r_1 = 0.26\alpha$ .

This dissertation contributes to the definition of new figure of merits concerning optimizing the photonic crystal waveguide design with respect to the FWM phenomenon. Simply achieving a large  $\eta$  is not always sufficient in many applications, since other important aspects need to be evaluated. For a given length  $L$  and pump power  $P_0$ , one should also be interested in the available bandwidth, which can be quantified in terms of the optical pump wavelength range  $\Delta\lambda$  in which does not fall below a certain level (say 3 dB) of its maximum value  $\eta_0(P_0, L)$ . Tunability is another important aspect that can be quantified as the wavelength separation  $\delta\lambda$  between the pump and the signal waves for which  $\eta$  is again higher than 3dB compared to  $\eta_0$ . We define the product of the maximum efficiency, bandwidth, and tunability ( $EBT$ ),

$$= \eta_0 \times \Delta\lambda \times \delta\lambda \quad (22)$$

A large  $EBT$  value should ensure a smooth wavelength dependence for  $\eta$ , which is important in wavelength division multiplexing systems. Also, since modern

trends in optical research dictate the use of compact, low-power components, we may also use a more powerful and size-aware FoM,

$$EBT_{PL} = \frac{\eta_0 \times \Delta\lambda \times \delta\lambda}{P_0 \times L} \quad (23)$$

Optimizing the waveguide with respect to  $EBT_{PL}$  is expected to yield shorter structures requiring less power at the expense of a smaller overall efficiency. We maximize the above FoMs based on an interior-point optimization method that combines a direct method for solving the constrained maximization problem, along with conjugate gradient steps using trust regions performed by MATLABs `fmincon` function. In our calculations, we assume a silicon PCSW with fixed  $\alpha = 412nm$ ,  $h = 0.5\alpha$  while the radii of holes not included in the optimization are also held fixed at  $r_\alpha = 0.27\alpha$ . We also assume that the range of the design parameters is  $0.04\alpha \Delta r_i 0$ ,  $0 \Delta y_i 0.15\alpha$  and  $0.1WP_02W$ ,  $25\mu m L 510\mu m$ . The photonic crystal waveguide design obtained by maximizing EBT FoM is obtained for  $\Delta y_1 = 0.149\alpha$ ,  $\Delta y_2 = 0.099\alpha$ ,  $\Delta y_3 = 0.012\alpha$ ,  $\Delta r_1 = 0.23\alpha$ ,  $\Delta r_2 = 0.24\alpha$ ,  $\Delta r_3 = 0.27\alpha$ , yielding  $EBT = 7.9nm^2$ . Similarly, the optimum photonic crystal waveguide design with respect to the  $EBT_{PL}$  FoM is obtained for  $\Delta y_1 = 0.151\alpha$ ,  $\Delta y_2 = 0.11\alpha$ ,  $\Delta y_3 = 0.013\alpha$ ,  $\Delta r_1 = 0.23\alpha$ ,  $\Delta r_2 = 0.27\alpha$ ,  $\Delta r_3 = 0.27\alpha$ , yielding  $EBT_{PL} = 73.9fm/W$ .

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# Using scripting languages for hardware/software co-design

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**Abstract.** In this thesis we present a new vertical methodology targeting the hw/sw co-design of embedded SoCs. For the suggested methodology a digital design and verification tool named System Python (SysPy) has been developed, using the strengths of the popular Python scripting language. We exploit the features of the language to boost the productivity of processor-centric SoC designs for Field Programmable Gate Arrays (FPGAs) implementation. In more details we developed methods to: (a) support hardware descriptions using Python syntax and automatically generate synthesizable VHDL code. (b) Support Python descriptions for simulating the behavior of an embedded SoC in algorithmic/functional level or using Register Transfer Level (RTL) descriptions and generate digital simulation plots in a bit-true and cycle-accurate manner. (c) Support the use of C software development tools for the programming of the processor core and (d) automatically generate Tcl scripts to integrate with FPGA implementation tools and ease synthesis and physical implementation steps. Complex SoC's have been designed and implemented in FPGA devices and used as design cases to demonstrate the features of the supported design flow. All designs use a processor IP core as the main programmable system controller, used for data processing. Each design follows the progress that we had in the development of our methodology and highlights certain features of the tool. We believe that with our methodology we cover the lack of existence of tools targeting the hw/sw co-design and prototyping of FPGA based embedded SoCs.

**Keywords:** Python, Processor-centric SoCs, hw/sw co-design, VHDL, FPGA, SysPy

## 1 Introduction

Modern Field Programmable Gate Array (FPGA) devices can host very complex digital designs. Most of the implemented System on Chips (SoCs) incorporate at least one programmable microprocessor (uP) unit. The processor's Intellectual Property (IP) core is key elements for the rapid prototyping of new digital systems, but on the other hand its usage raises a lot of design challenges that have to be addressed in the design flow.

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The main goal of this dissertation was the development of methods and of a design tool, called System Python (SysPy) targeting the hardware/software co-design and verification, using high-level abstract descriptions, of processor-centric SoCs implemented using FPGAs. For the needs of the research, we evaluated Python's programming features and especially the combination of scripting capabilities in a Linux shell, combined with Object Oriented Programming (OOP) features. These supported features could be used to:

- Implement high-level abstract models of blocks, e.g. arithmetic, memory and logic blocks, connect them using structural Python descriptions and translate them automatically to FPGA synthesizable Very high speed integrated circuit Hardware Description Language (VHDL), or use them to perform Register Transfer Level (RTL) bit-true simulation of a system. The integration of the SciPy library in Python provides a large number of functions which can be used for modeling arithmetic blocks.
- Build a framework and a design tool that implements the end-to-end design flow of a processor-centric system-on-chip, which invokes/calls other hardware and software related tools, e.g. logic synthesizers, software compilers, simulation tools etc.
- Process the large number of text-based files generated during a hardware design flow. Information extracted from generated text files is used many times as an input for the next design step or can be transformed/parsed to a different format.

While designing a complex digital system cannot be done automatically at a press of a button, we envisioned a design tool that would integrate the majority of the tools needed for an FPGA implementation of an embedded SoC. The first and most difficult task was to build the Python-to-VHDL parser. For this task we needed to define our supported coding style/syntax for the hardware descriptions in Python. The syntax should support a level of abstraction but on the other hand support features that are used in well established Hardware Description Language (HDL) languages, like VHDL and Verilog. A lexical analyzer also needed to recognize and track, in the user supplied Python descriptions, the supported syntax and parse these parts of Python code that later on would be mapped and translated to synthesizable VHDL.

The main contribution of this dissertation is to show that a modern programming language like Python can be used to design, simulate and implement processor-centric embedded SoCs, using high-level, abstract descriptions. This is very useful especially early in the design flow when control and processing logic of a system must be partitioned among software and hardware implementation. Our research work also shows convincingly that Python is a good candidate language to handle the large number of design tools needed to capture and implement a SoC in an FPGA device, in terms of hardware and software development.

## 2 Importance of processor-centric Systems-on-Chip

Most of the complex SoC designs implemented nowadays using FPGAs are processor-centric, where the uP is used as a gateway, implementing in software different communication protocols needed to exchange data with other logic devices on the same board or with a connected PC/server. Software executed by the processor is also used to control and exchange data with other custom blocks implemented in the FPGA fabric. Complex control and data processing logic can now be easily partitioned between software and hardware inside the same FPGA device. The latest devices from FPGA vendors, such as the Virtex-7 device from Xilinx or the Arria-V device from Altera, include fast embedded dual-core ARM processors which can communicate with the rest of the FPGA fabric using a very fast data and control bus. New designs can be efficiently prototyped within a few days, while the performance and power consumption of a SoC in an FPGA device can be now compared with that of an ASIC implementation. Of course for mass IC production the ASIC device remains the only option, especially in terms of cost reduction.

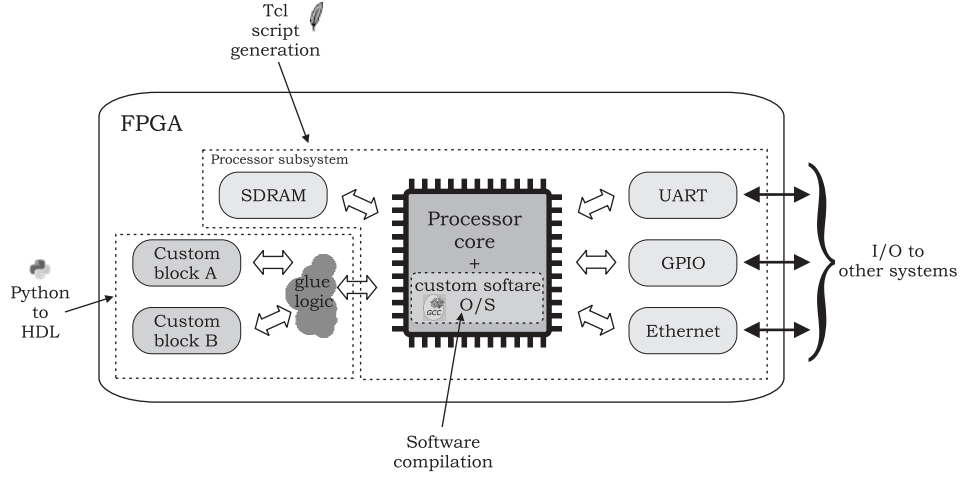
### 2.1 Python's innovative features used for digital design

For describing digital systems, choosing the proper language to develop the tool was of great importance. The special features that are required in a design flow targeting processor-centric designs and hw/sw co-design are:

- Integrate under the same scripting environment all the required tools for hardware/software co-development and co-simulation.
- Support clear and powerful syntax that could be used to describe hardware digital designs using HDL-like syntax or in an abstract way, using functions to auto-generate HDL code.

Figure 1 shows how we have used Python to handle in SysPy the integration of the ready-to-use processor IP-core in a SoC design. FPGA synthesizable VHDL code is generated from Python description for custom blocks along with data/control bus interface/glue logic. SysPy generates Tcl scripts for each one of the supported processor IP cores, which executes along with the FPGA synthesis tools in a command line, in order to incorporate the processor subsystem in the design. The tool also compiles, using the GNU Compiler Collection (GCC) C tools, the processor's control software along with any existing O/S kernel. FPGA bitstream file along with the compiled software binary files are generated, since SysPy makes all the necessary external tool calls (synthesizer, compiler), and can be used for FPGA implementation. In previous work [13] we have presented how SysPy can be used to provide Python level hardware descriptions to ease the design flow of processor-centric SoCs implemented on FPGA devices.

Python has been used for the development of commercial and widely used complex hardware and especially software development tool projects, where the text manipulation and generation features of the language are exercised in the



**Fig. 1.** Generic processor-centric SoC diagram.

best way. Earlier efforts using Python in hardware design include the following: *PyHDL* [1] allowed for structural descriptions which simplified system design using optimized hardware objects. *PHDL* [2] used Python to support a higher level of abstraction for hardware design. A designer can structurally describe a system using components from a Python library and parameters' selection can alter the size of a module, e.g. the bus width. Other tools used Python directly as an HDL. *MyHDL* [3] supports, as SysPy, behavioral, dataflow and structural hardware blocks design capture, and provides behavioral simulation functionalities, presenting text based simulation results. In addition, *MyHDL* can translate Python descriptions to either VHDL or Verilog. However, unlike SysPy, the above methods do not allow mixing Python with VHDL module descriptions in a design (all system components must be described in Python). *PyMTL* [4] targets digital hardware design by unifying functional, cycle-accurate and RTL hardware descriptions. The tool provides HDL generation, by translating Python descriptions to Verilog. Emphasis is placed on accelerating the functional and RTL simulation by converting Python testbenches to C++. However, unlike SysPy, the tool does not generate any files, e.g. Tcl scripts, folder hierarchy etc., to accelerate the FPGA implementation process by synthesis tools. Moreover, the simulation models do not include timing and latency information. Most important of all, none of the above described efforts focuses on handling processor IP cores and their software development environment. None of the above also supports creation of high-level verification models, in the way SysPy does, i.e. by using algorithmic descriptions mixed with RTL level descriptions.

In Table 1 a comparison of the Python related tools targeting digital hardware design is presented. All of the tools support Python code translation to RTL code, but only SysPy and *PHDL* support generation of FPGA synthesizable RTL code. Except from SysPy, only *MyHDL* supports behavioral simulation of



a designed system at the Python level, but no Value Change Dump (VCD) file generation is supported, which is the industry’s standard for storing simulation results. Except SysPy, none of the other tools support:

1. processor-centric designs and related C compiler tools handling for software development.
2. RTL code generation using parameterized Python functions.
3. abstract simulation of a design by mapping hardware functionality to Python function and classes.
4. hw/sw co-design using high-level hardware descriptions along with software expressed using C code
5. simulation and generation of VCD files for top-level I/O signal visualization.
6. automatic generation of Tcl scripts for FPGA synthesis tools.

Tools	Support for						References
	Python to RTL	FPGA synthesizable code	Behavioral simulation	Hw/Sw co-design	Processor-centric design	Synthesis tools integration	
PyHDL	X	-	-	-	-	-	[1]
PHDL	X	X	-	-	-	-	[2]
myHDL	X	-	X	-	-	-	[3]
PyMTL	X	X	X	-	-	-	[4]
SysPy	X	X	X	X	X	X	[5]

**Table 1.** Python digital hardware related tools comparison.

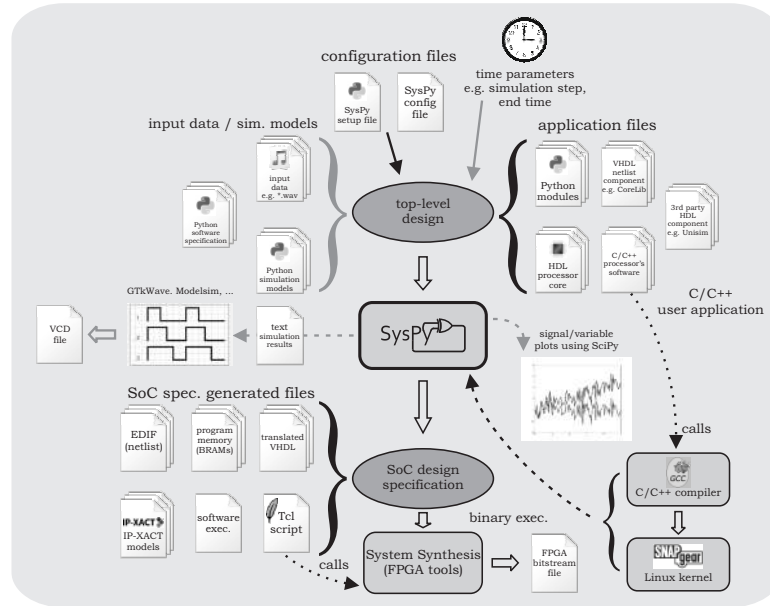
In all design and simulation steps of our design flow we use Python structures/syntax, and not any custom-defined syntax, to describe the datapath of the SoC and the related simulation models. All supported hardware description and simulation programming structures are compatible with the basic coding style used by the majority of Python programmers. In this way we tried to ease modeling and implementation of a processor-centric SoC even by software programmers with limited experience in hardware design.

### 3 SysPy digital design and verification features

In SysPy, Python acts as the backbone of a set of tools for hardware description as well as to incorporate other software tools. A typical design cycle starts by providing the simulation models of the desired system and also the timing information, e.g. main clock frequency, duration of the simulation, input data etc. The hardware description can have an HDL-like syntax supported by SysPy or a more abstract algorithmic-style syntax. The first syntax style can later be translated by SysPy to synthesizable VHDL code, while the later one cannot be directly translated to VHDL, but can help a designer to easily verify the functionality of a system.

All the main features of SysPy are presented in the supported design flow shown in Figure 2. The supported flow covers six major tasks related to the design of a processor-centric SoC:

1. Description in HDL of components (modules) that are going to be connected with a processor soft core.
2. Incorporation in a design of ready-to-use components and connection to a processor core.
3. Functional simulation of Python code describing the behavior of hardware blocks and of software executed by the processor
4. Generation of scripts for automating the software development flow for the processor core, e.g. automated calls to C compiler tools, initialization of the processor's program memory in BRAMs etc.
5. Generation and execution of scripts to automate the processes involved in a SoC's design flow, e.g. Tcl scripts for FPGA synthesis tools etc.
6. Generation of meta-data XML description of Python described IP cores, compatible with the IP-XACT standard [6].



**Fig. 2.** Processor-centric SoC design flow using SysPy.

To demonstrate the processor-centric design and verification features of our methodology, we present two large SoCs used as design cases to assess and improve our tool's features.

### 3.1 Biomolecular interaction networks simulation SoC design case

Advanced high performance computational techniques have been used the last decade in several scientific domains to ease and accelerate simulation of complex physical phenomena. Computational and systems biology is a rather new scientific field that takes advantage of computing techniques to describe and simulate complex biological phenomena. Stochastic simulations of biochemical reaction networks, called BioModels [7], [8] can be performed to study the properties of these biological systems. The main idea of the design of our SoC was to accept as an input BioModel files and simulate the chemical reactions of the described system using a stochastic simulation algorithm.

In [9] we have shown how using parallel processing and pipelining it is possible to design a core that performs efficiently, stochastic simulations of biochemical reaction networks (BioModels) based on the First Reaction Method (FRM) of Gillespie’s Stochastic Simulation Algorithm (SSA) [10]. Using SysPy’s processor-centric design and verification features, in [5],[14] we presented a flexible multi-processor SoC around the Leon3 processor and connected to the SSA core. In this design we proved how using SysPy we can automatically parse a BioModel file in SBML format, which uses XML syntax to define a biochemical reaction network. All the model’s important parameters are extracted and used to automatically construct the memory structures and the top-level specification necessary for synthesizing the SoC’s FPGA design. With this design approach the SoC can be used to process any BioModel of interest (captured in SBML) without any user intervention or required expertise in FPGA design.

Three different versions of the SSA core implementing the FRM algorithm have been designed, using one (FRM1X), two (FRM2X) or four PEs (FRM4X) operating in parallel ( $N=1$  or 2 or 4). The values of parameters  $m$  (number of reactions) and  $n$  (number of species) are automatically parsed from the BioModel SBML file, while the rest of the parameters are declared by the user at the top-level Python description. The mode parameter defines the SoC’s mode of operation.

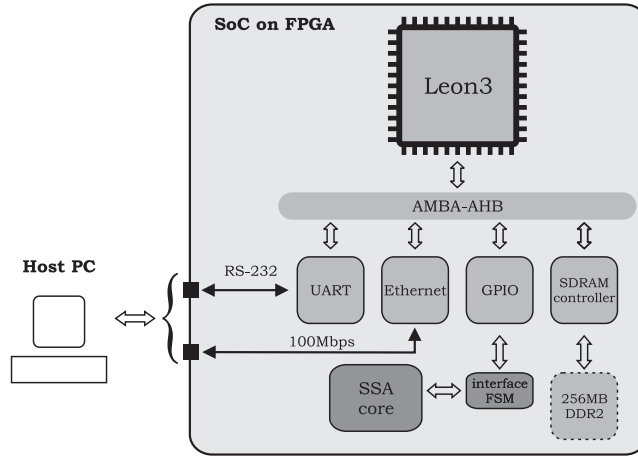
SSA core	Reaction Cycle time (us)	MReaction Cycles/sec.
FRM1X	1.356	0.737
FRM2X	0.93	1.075
FRM4X	0.73	1.37

**Table 2.** Throughput of the SSA cores at a clock frequency of 160MHz for a network with  $m = 136$  reactions and  $n = 93$  molecular species.

Using the features of our tool we managed to easily connect the SSA IP core to the Leon3 processor core. Leon is used to connect the SSA core to a host PC and also provides to the core access to a fast SDRAM DDR2 256MB

memory module. The SSA core is connected through an interface FSM which is attached on Leon's AMBA bus, along with all other connected peripheral devices. A top-level schematic of the SSA SoC is shown in Figure 3. For the implementation of the SSA SoC we used an ML509 Xilinx board equipped with a Virtex-5 XC5VLX110T-1 FPGA device. The board has also an SDRAM DDR2 256MB memory module clocked at 190MHz and a PHY Ethernet chip operating at 10/100/1000Mbps.

The implemented SoC delivers performance (0.353MReactions/sec) that is more than an order of magnitude higher compared to a computer cluster [11]. Since performance does not depend heavily on the specific BioModel used but on its complexity we conclude that a well designed FPGA SoC implementation can outperform cluster solutions for complex models. This is also not surprising since the FPGA parallel implementation does not suffer from any costly off-chip communication time overhead for distributing/collecting data during the parallel simulation.



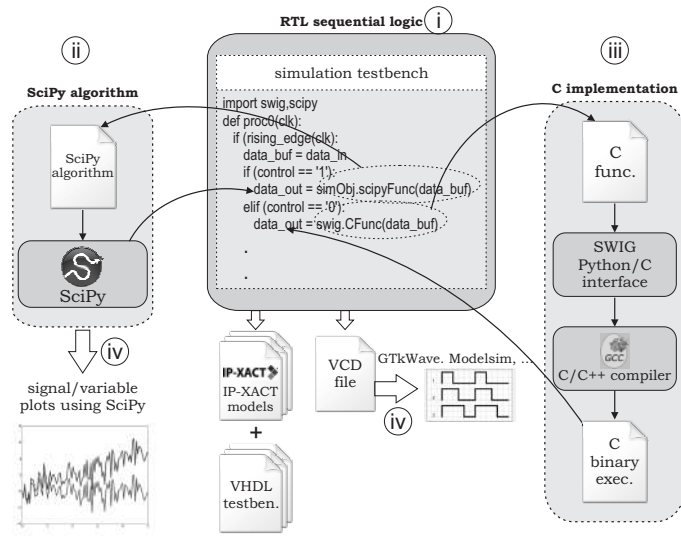
**Fig. 3.** SSA SoC. Connection of the SSA core to the Leon processor.

### 3.2 Audio processing SoC design case

Using the design of an audio signal processing SoC [12], we demonstrated the verification and O/S based software development flow supported in SysPy. A processor in an FPGA fetches audio files from a host PC through an FTP connection and analyzes the audio information using a hardwired FIR filter dividing the audible spectrum into four frequency regions. According to the filtered signal, the processor classifies a file into one of four music styles. The goal of SysPy's verification mechanism is to combine in the same testbench abstract algorithmic descriptions, RTL hardware descriptions and embedded software code for the processor core. It is possible to interchangeably use: a) SciPy (Matlab-like) algorithmic descriptions for arithmetic operations mapped in hardware and b) embedded C code executed by a processor core, to build bit-true and cycle-accurate

system-level verification models and generate digital waveforms to assess a SoC's behavior.

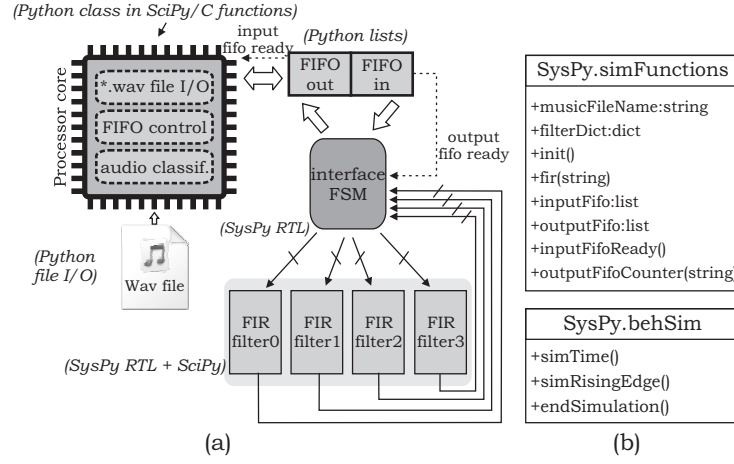
In Figure 4, a typical testbench in SysPy i) describes, using HDL-like syntax, the main elements of a pipelined datapath. ii) Python code in SciPy is used to describe in an algorithmic way functionality of hardware blocks not yet defined in HDL. iii) The same functionality can be expressed using C code, in case the required SoC functions need to be ported to software executed by a processor core. iv) Signals plots are generated during simulation in SciPy to observe signals behavior and also SysPy generates VCD files to represent the I/O signals of the system in binary format.



**Fig. 4.** Simulation flow in SysPy using RTL and algorithmic models.

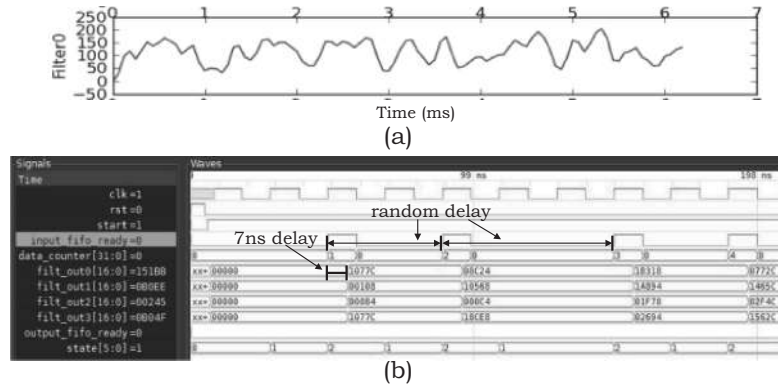
Using Python simulation models, we were able to make decisions regarding key aspects of the hw/sw design space, such as: a) filter properties e.g. fixed-point notation, number of taps, filter tap value and cutoff frequencies, b) data buffers size and control signals between the processor core and the filter bank and c) software running on the processor in a Linux kernel, that allows data to be handled in a file oriented manner. In Figure 5 a block level schematic of the simulation model is presented that reflects the partitioning of functionality between hardware and software. The text in parentheses designates the type of Python structure used to simulate each block. Software modules are represented using dashed line boxes inside the processors block, while the rest of the blocks correspond to hardware functionality. Software handles music file I/O, transmits the audio samples to the input FIFO, reads back the filtered samples and finally classifies the audio files by analyzing the filtered audio bands. Simulated

hardware functionality in the SoC involves the FIFO memories, the interface FSM handling the data traffic from and to the FIFO memories and the four FIR filters.



**Fig. 5.** a) Abstract modeling of the SoC using SysPy and SciPy, b) diagrams of the Python classes used in the SoC's testbench.

Two types of waveforms in SysPy can help a designer decide about system features early in the design flow: a) arithmetic plots in SciPy prove the correctness of an algorithmic model and b) digital signal plots in VCD format, where the variables of an algorithm in SysPy are converted (utilizing ready-to-use functions) to binary fixed-point format (bit-true) and plotted along with time information (cycle-accurate).



**Fig. 6.** a) Plots of the filtered audio signals using SciPy, b) waveform of the SoC's I/O signals using GTKWave.

In the waveforms, all the clock, reset, control and data signals, are presented. The filter outputs are presented after a user defined delay in the testbench (7ns). Also the `input_fifo_ready` control signal mimics the way the processor provides

audio samples to the input data buffer asynchronously for realistic simulation. We implemented the system using the ML509 Xilinx board with a medium size Virtex-5 FPGA device (XC5VLX110T-1), 256MB SDRAM DDR2 memory clocked at 190MHz, a PHY Ethernet chip and a serial RS-232 transceiver. Our SoC is a synchronous digital design, using a 90MHz domain for the filters' datapath and a 160MHz domain for the processor and the rest of the blocks. According to the specification, the logic synthesizer was able to synthesize the appropriate clock trees and reset circuitry.

## 4 Conclusions

In this work we demonstrated how a popular and freely available language like Python, can be used as a unified environment/platform to describe a SoC in a high abstraction level, verify it and deliver RTL FPGA-synthesizable code. Across all design and simulation steps in our flow we use Python structures/syntax, and not any custom-defined syntax, to describe the datapath of the SoC and the related simulation models. This is very important since the main target group of a high-level design tool are engineers and scientists who have little or no experience at all in digital hardware design. The goal in this case is to deliver a tool where a high-level interface can be used to:

- Design a SoC in a block-oriented way, using IP cores in RTL or netlist format and apply minimum effort to include any required digital glue logic between the blocks.
- Support a high-level verification flow, where Python descriptions can be used along with Matlab-like or C descriptions to simulate a digital block in a functional/algorithmic level and also in a cycle-accurate Register-Transfer-Level.
- Ease the use of digital synthesis and physical implementation tools for FPGAs, by auto-generating synthesis and compilation scripts.
- Provide tools to interface a SoC design after its implementation, in the form of software components running in parallel on the processor-core in the SoC and in the host PC connected to the SoC.

All four items in the previous list are critical in modern SoC designs. We believe that SysPy comes as an integrated environment and utilizes Python best programming practices like object oriented programming, text processing features, associative lists and ready-to-use numerical libraries, in order to design, verify, implement and test a processor-centric SoC. SysPy supports the most common and basic Python syntax and also any third-party tool or file format used or called within SysPy is adopted by the EDA industry and the software community tools, like Tcl, VCD, Linux OS, SciPy and gcc compiler. In this way our tool was implemented on top of already existing, popular and standard tools used in a hw/sw co-design flow and we do not introduce any new, custom defined and “exotic” practices that would be valid only in SysPy.

To get feedback from the community we provide SysPy as an open source tool through a public code repository [15]. Many code examples are provided along with information on how to setup the tool. This work has been supported by the Greek State Scholarships Foundation (IKY) under grant 2008-5530.

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# Complex patterns in telecommunications network planning

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**Abstract.** This thesis discusses a series of related issues raised towards the efficient telecommunications network planning, taking into account the existence of complexity. Three aspects of the associated complexity are examined, focusing on both the telecommunications networks per se, and the underlying street networks: (i) the observation of complexity, (ii) its effects and (iii) its utilization. More specifically, the empirical findings which demonstrate the existence of complex connection patterns are initially described. By collecting novel datasets of network data and graph-theoretically analyzing them, specific patterns of complex connectivity are identified. Next, the impact of complexity in the network efficiency is examined. The comparison of both synthetic and real-world topologies in terms of availability, congestion and cost, shows significant differences and indicates the criticality of specific topological properties. Especially, concerning the traffic congestion, the inclusion of topologies based on the gravity model reveals their superiority. Subsequently, a novel methodology is proposed, using synthetic complex graphs for the preliminary design of fiber-to-the-x telecommunications access networks. In particular, the utilization of planar proximity Gabriel graphs to represent the underlying layer in an urban environment is proved to be superior to the conventional geometric dimensioning models.

**Keywords:** complex networks, statistical network analysis, optical access networks, street networks, network planning

## 1 Dissertation Summary

### 1.1 Introduction

We live in an increasingly interconnected world in which infrastructures composed of different technological layers are interoperating. These infrastructures have progressively become crucial to our human society, as their connectivity has risen over time,

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and their emerging structure has been a hot research field to understand and handle. Examples are provided by the World Wide Web, the Internet, telecommunications networks, electrical power grids and street infrastructures as well.

Especially for telecommunications, it becomes of vital importance to study their features and incorporate the findings into better design and performance solutions. The ever increasing demands for communication and high bandwidth, caused by the introduction of new services and applications, as well as the growing number of users and devices connected to the Internet, pose new challenges for the telecom operators. A possible delay in the provision of telecommunications services (due to economic infeasibility) or uncertain disturbances of a telecommunications network constituent parts (due to failures or attacks) may affect a sizable proportion of the population and the economy, thus the recent focus on the telecommunications connection patterns can be well explained [1].

Most times these infrastructures can be considered as networks. A network, also called a graph in the mathematical literature, is in its simplest form, a collection of interacting constituents joined together in pairs by lines [2]. In the jargon of the field, the constituents are referred to as nodes or vertices and the lines are referred to as edges or links. However, most of the network properties cannot be understood by solely studying the properties of the individual components (nodes) or interactions (edges). Emergent features are coming out from the underlying *pattern of connections*. The appearance of patterns, that are hard to derive from the knowledge of the network elements exclusively, could be described by the term *complexity*. This term implies that the connection patterns may not be straightforwardly predictable; they could be neither purely regular nor purely random, but complex.

It should not be considered as a surprise that a particular pattern of connections, shaping the structure or topology of such networks, can strongly affect the behavior of the network functions. For instance, the pattern of connections between routers on the Internet, does affect the routes that data take over the network and the efficiency with which the network transports those data. Understanding, predicting and eventually utilizing or controlling the networks' non-trivial features is becoming a major intellectual and scientific challenge [3].

Comprehending how these *complex networks* organize and operate is mainly achieved by making use of the Graph Theory nomenclature and tools. Dealing with networks as graphs, and in order to characterize their connectivity, specific topological measurements are used [4]. In particular, the average node degree, the node degree distribution, the average path length, the diameter, the density, the clustering coefficient, and lastly the centrality measurements are the most basic metrics to express the relevant topological features. Of course, more advanced metrics can be used to support more sophisticated features, e.g., the assortativity, the algebraic connectivity, the entropy, the symmetry ratio, etc.

Although the study of complex networks has only recently become a major research issue, its origins can be traced back to the pioneering work on random graphs by Erdős and Rényi [5]. Modifying the Erdős-Rényi (ER) random graph model by using a rewiring procedure, almost 40 years later, Watts and Strogatz suggested a more realistic network formation model that produced graphs with *small-world* properties,

i.e., high clustering coefficient [6]. Subsequently, the suggestion of the *scale-free* networks, using the new concepts of *growth* and *preferential attachment*, emerged as the most popular formation model, also known as the Barabási-Albert (BA) model [7]. More recently, in 2007, Jackson and Rogers modeled another growing network in which new nodes chose their connections partly based on random choices and partly based on maximizing their utility function [8], also called the Jackson-Rogers (JR) model. Another network model that has lately experienced a resurgence of interest is the gravity model, rooted in Newton's gravitational law [9]. Beyond the deluge of formation models, a plethora of studies have been conducted to assist the understanding of dynamic behavior of real networks, which implies the processes running on top of network topologies [10]. Such studies attempt to uncover the principles behind many network kinds, including telecommunications, social, biological or street networks. Especially for the latter, the strong dependence of infrastructure network elements as well as their connections on the actual geography of the underlying street networks makes it of great importance for the engineering part of the infrastructure (e.g., telecommunications) to be aware of the street networks properties.

Nevertheless, there is plenty of space left for further advancement in the field, as the complex networks literature continuously provides contemporary insights. On top of that, exploiting ways in which the telecommunications networks knowledge and the features of the underlying street networks can jointly improve the telecommunications planning are still lacking.

The aim of this thesis is to discuss a series of related issues raised towards the efficient telecommunications network planning, taking into account the existence of complexity. Focus is set on both the telecommunications infrastructures per se, and the underlying street infrastructures. The ability to handle both infrastructures with graph-theoretic and complex network principles provides an ideal setting for observing and evaluating the impact of their complex patterns. In addition, this thesis contributes to the utilization of the complex characteristics of street networks in a telecommunications dimensioning problem. Explicitly, three facets of the associated complexity are discriminated and discussed, which consider both the infrastructure of telecommunications networks and the infrastructure of the underlying street networks: (i) the complexity observations, (ii) the complexity effects and (iii) the complexity utilization.

## 1.2 Complexity Observations

With regard to the technological networks and specifically the telecommunications networks, despite the engineer's dominant role in the case of the topology, unexpected and complex topological characteristics can appear. Empirical evidence [2, 4, 7], based on snapshots from the Internet and other telecommunications networks, has demonstrated the presence of power-laws and high robustness to random failures, though increased vulnerability to targeted attacks. In addition, it is not surprising that their topology is strongly constrained by their geographical embedding. Specifically, the distance dependence can soften the power-laws and increase the clustering coefficient, pointing to resemblance with small-world networks. Even though the telecom-

munications networks and the underlying street networks have not been jointly explored in the literature, there have been various studies empirically analyzing a series of street samples using the Primal approach [11]. What has been found is that street networks also possess a complex structure with a wide range of patterns [12-14], from tree-like to meshed connectivity. Universal properties have been observed among street networks, such as the values of the average node degree, the small average path length and the high clustering coefficient. Though, power-law node degree distributions in such planar networks have not been displayed, since the node degree is limited by the spatial embedding.

The current thesis set its focus on both the telecommunications networks per se, and the underlying street networks. Having a closer look, taking into consideration the empirical observation of telecommunications networks, a wide variety of measures, was examined over time, mostly related to the topological robustness [15]. As a result, it was found that only half of the considered measures have changed over time, even though the network order and size have tripled on average during that time period. As far as the measures that do change are concerned, nonetheless, not all of them are pointing in the same direction. For instance, the average clustering coefficient, the assortativity coefficient, the vulnerability and the natural connectivity, all exhibit improvement, while many other measures appear to worsen over time. Besides, towards understanding the complex structure of street networks, empirical observation was based on urban street networks and explored topological and geometric properties [16]. The outcome was that the number of edges, the topological efficiency and the total length are all correlated to the number of nodes, fitting well a power-law. Additionally, it was presented that the urban street networks of high population density tend to be relatively more “cost effective” compared to the low population density networks. In general, the street networks of a specific population density appear to have a perceptibly different behavior compared to those of different population density. Specifically, it is the number of nodes, the number of edges and the street length per person which have the strongest dependence on the population density. Moreover, several of the real-street properties were shown to display a satisfactory fit to planar proximity graphs, especially the  $\beta$ -skeleton graphs, thus providing a proficient technique to reproduce realistic street network properties [17].

### 1.3 Complexity Effects

Yet, the complex structure of networks does not come without effects. Apart from the inherent graph-theoretic interest, it is crucial to comprehend the behavior of the different complex topologies on which simulation or analytic studies are based. However, evaluating the impact of different topologies is not a new problem. In particular, a limited number of distinct network generation methods have been considered in previous studies in the context of network dynamics evaluation [10]. It has been deducted that the generation method indisputably has a significant effect on both the performance and the cost in each examined issue. With reference to the street networks, the effects of complexity have not been captured in the telecommunications planning

related studies, since the long-used geometric models typically assume a regular grid-like structure.

This thesis contributed to this direction through exploring the effects which non-trivial synthetic or real topologies have on related indicators, e.g., network availability, congestion and cost. Particularly, the comparison of topologies such as those based on the scale-free, the random, the gravity, and proximity graph models, under specific simulation settings made plausible the comprehension of differences among models and offered constructive design implications. Indeed, it was verified that the non-trivial structure of both telecommunications and street networks, influence the functionality of processes running over the topologies. More specifically, valuable knowledge was obtained on the way in which different complex networks can affect the optical network availability [18], mainly concluding that the average path length and the diameter have the most critical effect. Subsequently, it was discovered that the topologies created on the basis of the gravity model suffer less from congestion than the random, the scale-free or the JR ones, under either random or gravity traffic patterns [19]. Actually, the congestion level was found to be approximately correlated with the network clustering coefficient in the case of random traffic, whereas in the case of gravity traffic such a correlation is not a trivial one. Other basic network properties, such as the average path length and the diameter, were seen to correlate fairly well with the congestion level. In addition, a comparison among different gravity-based networks, as those have derived from different population distributions, revealed different cost behavior [20], i.e., from linear up to exponential. As well, the urban street complexity impact on the fiber-to-the-home (FTTH) cost was investigated [21] and striking differences among areas were found, which on the contrary the traditional geometric models would typically pass by.

#### **1.4 Complexity Utilization**

Beyond observing the complex characteristics and the influential effects of the related networks, a compelling challenge is to be able to utilize the derived knowledge towards more efficient communications [3]. Studies so far have insisted upon the importance of identifying successful resilience and survivability strategies. Basically, this has been possible by using the centrality measures to rank the importance of components which consist a complex network infrastructure. Furthermore, a number of mechanisms for topology control and improvement of network performance have been recently proposed, for instance, the socially inspired ones.

The contribution of the current thesis to the related literature was the suggestion of an innovative approach utilizing synthetic complex graphs in order to assist the telecommunications network planning [22]. Since it can be quite risky to rely on the conventional geometric models which regard the spatial structure of the road system simplified and regular, the idea of using a proper graph-based model which can incorporate the street complexity was supported. Since telecommunications networks are built on top of physically interconnected streets, it is plausible why the complex interactions of the underlying street level should be taken into account including application-specific requirements. Specifically, the use of the Gabriel graph model [23] as a

synthetic street network generator indicated its apparent sufficiency for the representation of realistic street structures. In addition, it was justified that the Gabriel model demonstrates a clearly distinct better fitting compared to the existing geometric models [24] on the fiber-to-the-x (FTTx) dimensioning and particularly on the early estimation of the main FTTx cost component – the trenching length. Besides, the conventional geometric models were verified to suffer from inaccuracy problems and thus could lead to wrong conclusion of the early techno-economic assessment. Especially in dense urban service areas, there were found striking differences between estimations and real-street calculations.

## 2 Results and Discussion

In this section, the two key contributions [19, 22] are described in more detail. Specifically, these are the congestion analysis of complex networks and the utilization of the Gabriel graph model for the FTTx dimensioning.

### 2.1 Network congestion analysis of gravity generated models

In relation to network congestion, most of the networks examined in existing studies were random (ER algorithm) or scale-free (BA algorithm) and the traffic flows have been assumed to be homogeneous between randomly selected source and destination nodes, with barely a few exceptions. However, in real networks, especially on the spatial ones, the topology can deviate from those derived from the ER or the BA models [25] and traffic is more likely to be generated/received unevenly at some nodes than at others, according to their characteristics [9].

Therefore, in the contribution [19], gravity topologies are introduced as they have been found to share statistical properties with real-world networks while allowing for optimal traffic exchange. Their behavior under congestion is compared to the behavior of random, scale-free and JR topologies. In addition, network congestion is studied taking into consideration traffic flows obeying the more realistic gravity-based flow patterns. The main purpose of this study is to examine the relationship between the complexity structure of different topologies and congestion factors under realistic traffic flows.

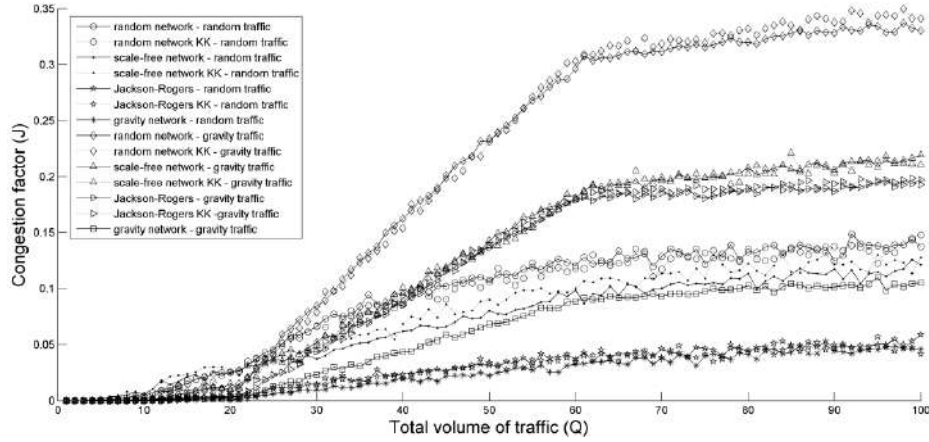
A simple model is applied, which considers traffic flows rather than packets, similar to that introduced in [26]. In that traffic model it is assumed that at each time step, unit traffic flow is generated between any two nodes belonging to the same connected component and capacities are randomly assigned on the links. Every origin-destination node pair  $i, j$  demands  $Q_{ij}$  traffic flows, presumably following different paths in the network. The capacity  $U_{ij}$  on the link  $(i, j)$  is randomly assigned in the range [20-60], which shows the maximum possible crossing flows on that link. Costs are also put as weights on the links using a special cost function, the US Bureau of Public Roads formula. The link cost is not a constant or a random value, but a function of the flows with congestion effects. Units of flows accumulate on the shortest path, and time step after time step congestion develops on it. Then, for subsequent

flows another path becomes the shortest in terms of travel cost, which again would become congested, and so on. Since all traffic flows in each time step are to be assigned simultaneously, a game is developed among traffic flows for the selection of the feasible paths with minimum cost (user equilibrium - UE conditions).

In addition to the standard paradigm of traffic exchange, the approach followed in that paper is again based on the gravity model perspective which allows for a realistic traffic generation for the entire network. Specifically, gravity-based traffic flows are generated in each time step, in the sense that every unit traffic flow is exchanged between node pairs, origin and destination, with probability defined by the gravity equation.

The random, the scale-free and the JR networks are separately considered with random node positions and with positions assigned by the Kamada-Kawai (KK) spring algorithm [27] in order to perform legitimate comparisons and alleviate potential concerns about the networks' equity against spatial advantages.

The congestion factor  $J$  is defined as the percentage of congested links out of the total links, as introduced in [26]. Obviously,  $J=0$  corresponds to uncongested traffic on the network, and  $J=1$  indicates the worst case of network congestion. The total network cost  $TNC$  includes the link length as a cost parameter, which is important in the case of spatial traffic networks. Specifically, it is defined as the summation of all links of the product of the three variables link flow, link cost and link length.



**Fig. 1.** Congestion factor as a function of total volume of traffic for random, random KK, scale-free, scale-free KK, JR, JR KK and gravity networks for both random and gravity-based traffic patterns<sup>1</sup>

<sup>1</sup> Each curve corresponds to an average over 20 independent realizations of the networks with 100 nodes and average node degree equal to 6. It is assumed for all networks' fitness distribution:  $\gamma=1$ , for the construction of gravity networks:  $\phi=1$ , and for the gravity-based traffic flows:  $\phi=1$ .

Both the standard paradigm of uniform traffic volumes between randomly interacting node pairs (random traffic) and the more realistic gravity-based interactions (gravity traffic) are depicted in Figure 1. In this figure, the parameter  $\phi$  is a constant representing the distance sensitivity and the parameter  $\gamma$  is another constant, for which the higher its value is, the more uniform in fitness the nodes are.

It is shown that depending on the traffic pattern, the networks have different tolerance to congestion, with the gravity traffic causing more severe congestion to all networks. Moreover, the study demonstrates that the topologies created on the basis of the gravity model suffer less from congestion than the random, the scale-free or the JR ones, plus at a lower cost. Furthermore, the congestion level is found to be approximately correlated with the network clustering coefficient in the case of random traffic, whereas in the case of gravity traffic such a correlation is not a trivial one. Other basic network properties such as the average path length and the diameter are seen to correlate fairly well with the congestion level. Further investigation on the adjustment of the gravity model parameters indicates particular sensitivity to the traffic congestion, whereas only minor sensitivity to the total network cost.

These findings may be explained by the structural properties of the gravity networks; they are neither uniform, nor power-law distributed. Although gravity networks can reproduce a scale-free behavior under particular circumstances, on the other hand the spatial constraints can make the network more homogeneous. Particularly, the distance factor can shape the node degree distribution to deviate from the power-law form [9, 25], allowing for a more distributed flow exchange. This is additionally confirmed by the statistical properties of gravity topologies indicating small average path length, small diameter and high clustering coefficient, respectively.

## 2.2 Incorporating Gabriel graph model for FTTx dimensioning

The study [22] supports the idea that using of a graph-based model, a better alternative street layout could be succeeded, in comparison with the imprecise geometric models and the costly area-specific Geographic Information System (GIS) solutions. Particularly, the utilization of Gabriel graphs [23], shown to incorporate the complexity of real street networks, is suggested as a novel credible abstract approach that conjointly meets accuracy, simplicity and generality.

Especially, these graphs are useful in modeling graphs with geographic connectivity that resemble grids, but additionally incorporate more complex traits. They have already been suggested as capable to capture the structure of telecommunications networks in the physical backbone level. Nevertheless, it is quite intriguing to examine whether the Gabriel graph can capture the structure of the urban street network – the basis of a telecommunications access network.

In the Gabriel connection scheme, two nodes are directly connected if and only if there are no other nodes falling inside the circle associated with the diameter that has the two nodes as endpoints. Mathematically, two nodes  $i$  and  $j$ , from a set of  $V$  nodes, are connected if the square of the distance between them is less than the sum of the squared distance between each of these points and any other point  $k$ . An undi-



rected graph is constructed by adding edges between nodes  $i$  and  $j$  if for all nodes  $k$ ,  $k \neq i, j$ , where  $d$  expresses the Euclidean distance:

$$d(i, j)^2 \leq d(i, k)^2 + d(j, k)^2 \quad (1)$$

Based on the Gabriel graph, the network installation would follow all streets and connect buildings (subscribers) which are all assumed to be distributed along the streets (graph edges) for simplicity reasons. The total length can be derived from simulations which only require the parameter  $|V|$ . It is thus necessary for the length estimation to be able to calculate the number of intersections  $|V|$  i.e., counting road intersections on the map.

Towards the analysis of the structural properties of Gabriel graphs, several properties are observed and compared to real-street datasets from various countries. More specifically, the dataset (100 Greek cities) and the procedure described in [16] are followed; however, three additional datasets – provided in related empirical studies – are supplementary taken into account [12-14]. Their samples are collected from diverse areas around the world (20 world cities, 118 US urban areas and 21 German cities, respectively) and correspond to large surface areas. For comparison reasons, all calculations refer to data represented using the Primal approach [11] and normalized (scaled down) to correspond to 1-square-kilometer area.

Moreover, by making use of the Mean Absolute Percentage Error (MAPE) statistic, it is demonstrated that the Gabriel graph model is able to produce synthetic networks quite similar to the considered real-street networks. In particular, the derived MAPE, for all the datasets and the basic statistical properties, is up to the level of 25%-30%. Regarding the crucial property of the total length, it appears to take values as a function of the number of nodes (intersections)  $|V|$ , in all real-street datasets, as well as in the synthetic Gabriel graphs. Despite the fact that the examined real-street samples represent very diverse geographic cases, the relation between the total length and the number of nodes can fit well a power-law, i.e.,  $\sim |V|^p$ . In the case of the synthetic Gabriel graphs, this particular equation, estimating the total length using solely the number of nodes, is:

$$W = 1.212 \cdot |V|^{0.572} \quad (2)$$

With respect to the estimation of the trenching length, a case study is presented concerning the fiber-to-the-building (FTTB) or FTTH deployment in the selected dataset of 100 urban areas in Greece, for which all required data are readily available (e.g., building density per sample area). Particularly, five traditional geometric models along with the Gabriel graph model are applied in order to estimate the required trenching length of the network installation and then compare results with the real-street lengths.

Specifically, the five most well-established geometric models for abstracting the fixed access network deployment area are the Simplified Street Length (SSL), the Street Length (SL), the Double Street Length (DSL), the SYNTHESYS, and finally the

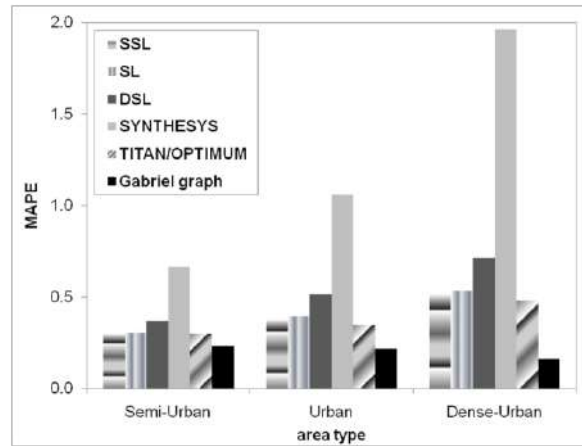
TITAN/OPTIMUM models [24]. In practice, they are used as a starting point to design the fixed street-based (buried) telecommunications infrastructure connectivity. They are commonly based on a set of parameters such as the customer and building density, the area size, as well as the average distance between end-users and the Central Office (CO).

**Table 1.** Models' fitting to the real-street trenching length

	SSL	SL	DSL	SYNTHESYS	TITAN/ OPTIMUM	Gabriel graph
MAPE	0.415	0.429	0.562	1.326	0.389	0.201

After this process, remarkable variances among the results of the simple geometric models can be clearly observed, which in turn lead to errors higher than 38.9%, as seen in Table 1. The numerical results indicate that the Gabriel model solution outperforms the existing solutions by the conventional geometric ones. Even more significant is the observation that the proposed Gabriel model leads to an approximately 20% error, that is to say at least 48% better accuracy (up to 85%) than any of the geometric models. Hence, the models may be ranked from the best to the worst as follows concerning the estimation of the trenching length: Gabriel graph, TITAN/OPTIMUM, SSL, SL, DSL, SYNTHESYS.

In addition, it is achievable to quantify the inaccuracy of the considered models, caused by the irregular street connectivity, discriminated in different area types: Semi-Urban (250-500 buildings/km<sup>2</sup>), Urban (500-1,000 buildings/km<sup>2</sup>), and Dense-Urban (more than 1,000 buildings/km<sup>2</sup>). Once again, it is easily apparent that the Gabriel approach offers much higher accuracy than geometric models do, especially in highly populated areas, where the existing models diverge even more from the real data (see Figure 2).



**Fig. 2.** MAPE behavior under different area types: Semi-Urban (250-500 buildings/km<sup>2</sup>), Urban (500-1,000 buildings/km<sup>2</sup>), and Dense-Urban (more than 1,000 buildings/km<sup>2</sup>)

### 3 Conclusions

Complex patterns of connections (i.e., power-laws and scale-free structure) have recently emerged in the network analysis studies, forcing researchers to finally depart from the assumption of random or regular topologies.

Particularly, this thesis contributed on the complex networks analysis and modeling, on the investigation of the emergent complexity effects, and on the development of methodologies to utilize complexity towards more efficient telecommunications network planning. Specifically, the introduction of time was proved essential in the complex network analysis, the coupling between street network complexity and population density was displayed and the  $\beta$ -skeleton graphs' sufficiency to reproduce real-street properties was shown. As well, the impact of different complex topologies to availability, congestion and cost was presented, the superiority of gravity topologies on congestion and cost was revealed, and finally the use of Gabriel graphs was proposed, indicating a departure from the conventional geometric models.

Although the above contributions provide significant new insights into the complexity met in both the telecommunications and the underlying streets, future research is needed in order to investigate more in depth aspects which were not included in the present thesis. In particular, the consideration of functional network characteristics would be the most relevant and practical supplement from the telecommunications services perspective. Besides, a more exhaustive observation of the gravity network statistical properties would unveil the explanation for their dominance under congestion. With regard to the Gabriel graph-based approach for the preliminary FTTx dimensioning, there still remains space for exclusively ascribing the topological and geometric metrics to engineering metrics and the calculation of the rest of the FTTx cost factors, i.e., fiber length, number of splitters and so on.

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# Methodologies for Effective Self-adaptive Decisions in Routing, Signaling, Cooperation, and Other Operations in Diverse Mobile Networks

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**Abstract.** In this PhD dissertation we provide methodologies for effective decision making in the fundamental networking functions in diverse network topologies. This provision becomes a difficult task, because the nodes belonging in such networks must take decisions based on several sources of context, related to the local environmental and nodal characteristics. The thesis fills this gap, by introducing quantities expressed in terms of time, which are the ‘*retaining time*’ and the more general notion of ‘*Decision-Related Event Occurrence Time*’ (DREOT). These novel notions are able to appropriately “translate” the environmental conditions and nodal characteristics (which may be adjusted, to also reflect social aspects through the novel notion of ‘*subjective density*’) in a form suitable for taking the decision in question. Furthermore, the notion of the retaining time has been successfully exploited towards dynamically self-adjustable routing in diverse mobile topologies by the proposed MAD (Maximum Advance Decision) protocol. The thesis studies also on-demand beaconing techniques (required for a realistic implementation of the routing protocol), suggesting a generic analysis and policies for information exchange between the involved nodes. A rich set of numerical and simulation results demonstrate the efficient and effective nature of the proposed techniques and notions.

**Keywords:** context aware decisions, network mobility, routing, beacon scheduling mechanisms, cooperation.

## 1 Introduction

Modern deployments of Mobile Ad Hoc Networks (MANETs) frequently exhibit topologies with highly variable characteristics, for example in terms of the nodal density and mobility. This is particularly true for Delay Tolerant Networks (DTNs), Wireless Sensor Networks (WSNs) and vehicular networks (VANETs) where the density and mobility vary through time and by location. These variable

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characteristics are highlighted even more in many emerging wireless applications with a need for network inter-operability. Moreover, the need for energy saving techniques, lead also to a further variability of the nodal density with time. Finally, higher layer aspects (e.g., social relations) might introduce additional nodal dependencies that can increase the variability of conditions even further.

For networks exhibiting the variable environmental conditions just mentioned, the provision of fundamental networking functions (such as self-organization or routing) becomes a difficult task, because the network nodes must take decisions based on several sources of context, related to the local environment and to the characteristics of individual neighbor nodes. The issue is typical in routing applications, where a node carrying a message is called to decide whether or not to forward the message to a neighbor node when an opportunity arises.

Because of the problem's complexity, most of the current routing protocols address only a subset of the possible conditions. For example, the forward-based multihop routing protocols [11] are usually employed for networks with a relatively high nodal density. On the contrary, in sparse topologies carry-based multihop protocols [3] fit better. However, none of the approaches just mentioned are capable of addressing the entirety of conditions that may be encountered. To combat this lack of adaptivity, other approaches try to take advantage of the multiple copies of the same message. However, the multicopy approaches achieve low delay and improve the reliability of message delivery [9], they are inappropriate for settings with scarce networking and device resources, due to their inherent redundancy.

The thesis targets the amelioration of single copy approaches. It introduces a routing protocol, called *Maximum Advance Decision* (MAD), based on the novel notion of the '*retaining time*', which is capable of automatically adjusting for best performance under the conditions applicable each time [6, 7]. Along the process, the thesis develops techniques that are useful for studying more general times between events governing a decision-driven system, called '*Decision-Related Event Occurrence Time*' (DREOT). Besides being useful on their own right, these results enable an in depth study of how the retaining time is affected by relevant network and nodal parameters and lead to an even grater improvement of the routing protocol [5].

It is noted that the significance of taking the right routing decision becomes more important in networks where social ties of varying strength are built between nodes. Proposals for forwarding strategies in such environments introduce the social tie as an additional parameter multiplying or adding to a purely contact-based metric [4] and failing to capture that these attributes largely shape, in turn, the topological characteristics of the network environment. The results in this thesis suggest a simple, yet non-arbitrary and robust, way of capturing the effect of social ties, through an appropriate adjustment to the notion of nodal density, called '*subjective density*' [7].

Another important complementary issue is the operational aspect of the signaling required for a realistic implementation of the routing protocol. The locally aware routing protocols base their next-hop selection on information about their

immediate neighborhood, gathered by means of a beaconing mechanism. In general, beacons may be proactively broadcasted from nodes to their neighbors (‘receiver-initiated’ beaconing) [10] or may be solicited by the node carrying the routed message (‘on-demand’ beaconing) [2]. On-demand beaconing is of growing importance, mainly in more dynamic and sparse environments, or in mobile environments with variable conditions, which is the main target of the thesis. The major question that arises in this context is: How frequent should beacons be? The thesis explores quantitatively the trade-off between beacon intervals (periodic and adaptive) and routing effectiveness [8].

## 2 Dynamically Optimized Routing for Networks with Diverse Density and Mobility Characteristics: the Retaining Time and the MAD Protocol

### 2.1 MAD: A Dynamically Adjustable Hybrid Location and Motion-based Routing Protocol

The Maximum Advance Decision-MAD protocol [6, 7] addresses a mobile ad hoc setting, where each mobile node can communicate directly with all other nodes within some given range  $r$ , defining the node’s neighborhood. By means of a beaconing process, the node possessing a message (called the current node) checks its neighborhood and evaluates the appropriateness of the neighbor nodes as a next hop for the message, according to the *advance* metric.

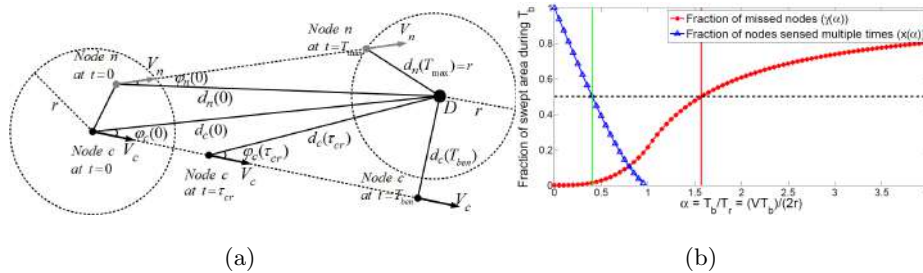


Fig. 1: (a) Quantities relevant to the MAD policy checks. (b) Fractions of nodes missed and sensed multiple times vs the  $\alpha$  parameter, and corresponding bounds.

When the current node  $c$  considers its neighbor  $n$  as a next hop candidate, it takes into account the anticipated progress due to both the forward and carry actions. The overall anticipated progress occurs over a time interval of length  $T_n$ , called ‘retaining time’ and is the time that this node would retain the message if it is selected as the next hop node. The advance metric is defined as the overall progress per unit of time due to receiving and holding of the message by node  $n$  and is given by

$$\text{ADV}_{cn}(T_n) = \frac{d_c(0) - d_n(T_n)}{T_n},$$

where  $d_i(t)$  stand for the distance of node  $i$  from the destination  $D$  at time  $t$  (see Fig. 1a).

This metric expresses the anticipated rate at which the message will approach its destination if node  $n$  is selected as the next hop. Such a selection will be reasonable only if this rate is higher than the one achieved when the current node retains the message. In order to determine this, the current node also calculates its own advance metric  $\text{ADV}_{cc}(T)$  over a common value of retaining time denoted as  $T$ , whose value is related with  $T_n$  and  $T_c$ . The relative merit of the neighbor node  $n$  over the current node  $c$  is expressed by the difference of the corresponding advance metrics

$$\Delta_n(T) = \text{ADV}_{cn}(T) - \text{ADV}_{cc}(T) = \frac{d_c(T) - d_n(T)}{T}.$$

The same procedure is repeated for all neighbors of the current node and ultimately the message is forwarded to the node with index  $j = \arg \max_n \Delta_n(T)$ , justifying the name of the protocol.

## 2.2 The Retaining Time Estimation and its Dependence from Topological and Node-oriented Characteristics

In order to compute the advance metric, the current node collects (by means of a beaconing process) information about the positions and velocities of its neighbor nodes. Given a neighbor's distance and velocity at time  $t = 0$ , the current node estimates the corresponding retaining times for all nodes  $n$ , as well as the node's distance at future time instants. In the absence of information about the future pattern of motion, MAD assumes that the neighbor will maintain straight line motion at constant speed equal to  $V_n = V$ , up to that future time instant. Also, let  $V_n$  and  $\phi_n(t)$  to denote the magnitude of node  $n$ 's velocity and the angle between this velocity and the line segment from node  $n$  to the destination  $D$ , respectively, at time  $t$  (see Fig. 1a).

It is clear that the notion of the retaining time should reflect the potential of the node as a message carrier, for the given nodal and environmental attributes. The original heuristic approach, has been presented in [6]. It follows an approach based on the fundamental notion that when node  $n$  moves towards the destination, i.e.,  $V > 0$  and  $\cos \phi_n(0) > 0$ , it remains a beneficial carrier for the message's routing until it reaches the point closest to the destination along its straight-line trajectory. The time required for node  $n$  to reach that point is equal to  $T_{\text{ben}}(d_n(0), \phi_n(0))$  (see the lower right part of Fig. 1a). The node shouldn't keep the message further, because after this time it will start moving away and its distance from the destination will keep increasing. In the complementary case, i.e.,  $V = 0$  or  $\cos \phi_n(0) \leq 0$ , node  $n$  should find a better next hop to forward the message as soon as possible, because the motion is harmful for the message's routing. The more neighbors around node  $n$ , the easier it becomes to find a next hop, thus the time required for that is a decreasing function of the network's nodal density  $\rho$  in the local environment of the node. In view of



these comments, the retaining time value is chosen so that the highest expected node density corresponds to the smallest possible value of the retaining time, equal to  $T_d$ . Putting together these observations, the original MAD estimates the retaining time as  $T_n = T(d_n(0), \phi_n(0))$  where

$$T(d, \phi) = \begin{cases} T_{\text{ben}}(d, \phi), & \cos \phi > 0, \\ T_d \rho_{\text{max}} / \rho, & \text{otherwise.} \end{cases} \quad (1)$$

One direction for improvement over (1) is to involve both of the density and speed parameters in the retaining times relevant to all directions of motion. Additionally, a node moving towards the destination can be treated in a more refined way, by distinguishing between directions enabling the node to approach the destination close enough for delivering the message itself and directions for which such delivery is not possible. Specifically, when  $\cos \phi_n(0) \geq \sqrt{1 - [r/d_n(0)]^2}$ , the destination will enter the node's range and receive the message at time  $T_{\text{max}}(d_n(0), \phi_n(0))$  (see upper right part of Fig. 1a). This is the greatest possible value for the retaining time in this case. A refinement along these lines was presented in [7]. Putting together the above observations, the improved MAD estimates the retaining time as  $T_n = T(d_n(0), \phi_n(0))$  where

$$T(d, \phi) = \begin{cases} T_{\text{ben}}(d, \phi) \frac{\rho_{\text{min}}}{\rho} + \frac{r}{V} \frac{\rho_{\text{max}}}{\rho}, & 0 < \cos \phi < \sqrt{1 - [r/d]^2}, \\ T_{\text{max}}(d, \phi) \frac{\rho_{\text{min}}}{\rho}, & \cos \phi \geq \sqrt{1 - [r/d]^2}, \\ \frac{r}{V} \frac{\rho_{\text{max}}}{\rho}, & \cos \phi \leq 0. \end{cases} \quad (2)$$

Both of the approaches for the determination of the retaining time have desirable properties and they can help MAD achieve self-adaptation in a considerable range of density and mobility conditions (see Section 3.1).

### 3 Methodologies for Calculating Decision-Related Event Occurrence Times

The notion of the retaining time appropriately “translate” the environmental conditions and nodal characteristics in a form suitable for taking the decision in question. Clearly, the retaining time is a particular instance of the more general notion of ‘*Decision-Related Event Occurrence Time*’ (DREOT) [5], which refers to the time duration up to the occurrence of an event linked to the decision, e.g., the time that an entity will sustain a property, or the time during which some conditions remain in effect, or the time until a node reaches a battery depletion level, etc.

Determining the value of a DREOT metric is challenging, especially in network environments with variable conditions. For a specific example, consider routing and the use of the retaining time for determining when (and to which node) to forward the message. In this setting, the node currently holding the message employs the retaining time pertaining to itself and to the other nodes

in contact with it, to take the forwarding decision. At the same time, a message forwarding decision marks the end of the retaining time for the current node.

Towards addressing these difficulties, the thesis contributes techniques for enabling DREOT-related calculations. The proposed methodology introduces “bounding” events,  $R_{\pm,n,t}$ , that are either broader or stricter than the events triggering the decision and are more amenable to analysis. Probabilistic reasoning about these more general events leads to bounds for the probability distribution and for the mean value of the DREOT. By employing some further, very mild, uniformity assumptions about the local environment, the results take a more concrete form that explicitly captures the nodal density and the degree of mobility and given by

$$T_-(0, \infty) \leq \mathbb{E}[T] \leq T_+(0, \infty),$$

with

$$T_j(t_1, t_2) \triangleq e^{-\rho\pi r^2 h_j(0)} \int_{t_1}^{t_2} e^{-\rho 2Vr \int_0^t h_j(\xi) d\xi} dt, \quad j = \pm$$

and

$$h_j(t) \triangleq \begin{cases} 1 - \hat{\omega}_-(t), & j = -, \\ \hat{\omega}_+(t), & j = +. \end{cases} \quad (3)$$

It is noted that it is always possible to use any lower bounding functions  $\hat{\omega}_{\pm}(\cdot) \leq \omega_{\pm}(\cdot)$  in place of  $\omega_{\pm}(\cdot)$ , and thus (3) has been written so as to indicate this possibility. The actual policy descriptor probability bounds  $\omega_{\pm}(\cdot)$  given by  $\omega_-(t_A, t_B) \triangleq \min_{n \in N(t_A, t_B)} \Pr\{R_{-,n,t_A}\}$  and  $\omega_+(t_A, t_B) \triangleq \min_{n \in N(t_A, t_B)} \Pr\{R_{+,n,t_A}^C\}$ , where  $N(t_A, t_B)$  is the set of nodes checked in  $(t_A, t_B]$ . The methodology and results just outlined are generally applicable, not being tied to a particular protocol or mobility model.

### 3.1 Decision-Related Event Occurrence Times in the Context of Routing: The Retaining Time

The precise nature and effect of the decision, at the abstract DREOT-related results presented in Section 3, is abstracted away to, so called, ‘*policy descriptor probability bounds*’. Expressions for these quantities can be derived by employing further properties of the specific policy and mobility model in hand. This is demonstrated by applying the general results to the particular case of routing according to the MAD protocol. The nodes are assumed to move according to the Random Direction Mobility model, which is particularly challenging with respect to the routing effectiveness. As already mentioned in Section 2, MAD bases its decision policy on the notion of the retaining time, so it can be readily addressed within the scope of the general DREOT framework. The calculation of the retaining times by means of the general DREOT-related results [5], as pursued here, not only leads to much more refined expressions, compared with the two variants of the heuristic expressions mentioned in Section 2.2, but also

provides formal justification for certain functional relationships and a reinforced insight, without the need for resorting to heuristics. Numerical and simulation results illustrate that the refined expressions for the retaining time make the protocol even more effective in adapting to a very wide range of mobility and density conditions encountered in real-world environments.

The simulations addressed a network topology where the mobile nodes moved within a rectangular open area of size  $10\text{km} \times 10\text{km}$ . The source and destination were static nodes at diagonally opposite corners of the rectangle; the destination's position was globally known. Mobile nodes could communicate with other nodes in a range  $r = 250\text{m}$ . These network parameters are also obtained for the set of simulation in the other sections.

The value of  $T_{\text{check}}$  and  $T_d$  was set equal to 1s and 0.2s respectively. The corresponding values reported here were obtained as averages over 1000 messages and the mobile nodes moved according to the random direction mobility model with zero pause time and constant speed [1]. In this set of results, we compare the performance of the MAD routing protocol where all the nodes take decisions using the minimum between the lower bounds of the retaining time for the current and the checked neighbor, with four other protocols. From the forward-based routing protocols we choose the greedy MFR protocol [11], which selects the candidate node maximizing the distance between the current node and the candidate node's projection point on the line connecting the current and destination nodes. A typical representative of the protocols focusing on the carry action is the MoVe protocol [3], which uses a metric based on the direction of the nodes' motion. The third is the MAD protocol with the original expressions for the retaining time [6] (labeled 'original MAD'). The last is the MAD protocol with the improved retaining time estimation [7] (labeled 'improved MAD'). A wide range of node density values were examined, ranging from 0.5 neighbors to 17 neighbors. For each value of nodal density, the speed value was examined corresponds to a very slow-changed topology with 10 km/h for each node.

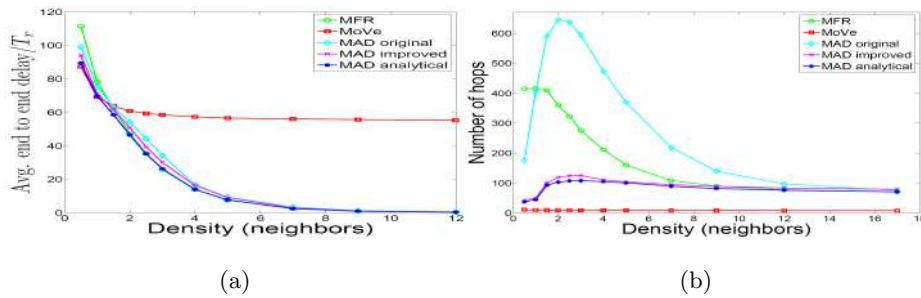


Fig. 2: Average (a) end-to-end delay and (b) number of hops vs nodal density for slow-changed topology (nodes' speed 2.8 m/s).

From the total results in Figs. 2a–2b, it can be directly evidenced that both of the average end-to-end delay and the number of hops to delivery (for all examined routing protocols) decrease as the nodal density increases. This happens because

a more dense topology results in an increased number of forwarding opportunities observed by the current node per unit of time. By comparing the end-to-end delay of the MFR and MoVe, the results indicate that the MoVe protocol outperforms in the sparse topologies and the MFR outperforms in the more dense topologies. The results indicate that all versions of MAD are capable of adjustment as the density increases. Also, for all the three versions of MAD protocols, the number of hops is lower in sparse topologies compared to that in dense topologies, because in sparse (resp. dense) topologies they tend to carry-based (resp. forward-based) protocols. The ‘analytical MAD’ achieves better efficiency regarding to the end-to-end delay and the number of hops than that of the ‘original MAD’ and the ‘improved MAD’. Specifically, the efficiency of the ‘analytical MAD’ is equivalent or better than that of the MoVe and the MFR protocols in sparse and dense topologies, respectively. This is because MAD simultaneously takes advantage of both the forward and carry actions.

#### 4 Exploiting Topology and Behavioural Attributes for Effective Routing in Mobile Networks

The number of forwarding opportunities observed by a node at each point in time is a key parameter influencing the node’s retaining time and, through it, the efficiency of the routing protocol. Up until now the forwarding opportunities have been quantified through the topological density. However, the forwarding opportunities observed by different nodes in an area may not be the same, due to different individual characteristics of the nodes inherited from higher layers. In doing so, the thesis introduces the novel notion of ‘*subjective density*’ [7]. This notion is defined as

$$sd_i = c_i \rho,$$

which adjusts the topological density according to higher layer properties relevant to individual nodes, such as cooperation attributes. The cooperation level, denoted as  $c$ , is a real number in the interval  $[0, 1]$ , where 1 indicates complete cooperation and 0 indicates complete lack of cooperation. Once these aspects have been captured, they can be incorporated in the decision policy of the routing protocol, in a simple yet robust way.

We compare two versions of improved MAD. In the first version (labeled “plain improved MAD”), the protocol uses the topological density ( $\rho$ ), while in the second version the protocol employs the subjective density ( $sd$ ) instead (this version is labeled “cooperation-aware MAD”). We also use a topology of moderate density (with 500 nodes in the rectangular area) and intense mobility (each node moves with speed 120 km/h). Mobile nodes moved according to the random way-point mobility model with zero pause time and constant speed [1]. Various environments of widely varying degrees of cooperation were examined, from completely cooperative to 90% of the nodes non-cooperative, varying the percentage of the total nodes that are non-cooperative in incremental steps of 10%. A node is characterized as cooperative if the value of its cooperation level

(c) ranges between 0.8 to 1, the actual value being sampled uniformly in this range. Similarly, a node is characterized as non-cooperative, if the value of its cooperation level (c) ranges between 0 to 0.2, the actual value again being sampled uniformly.

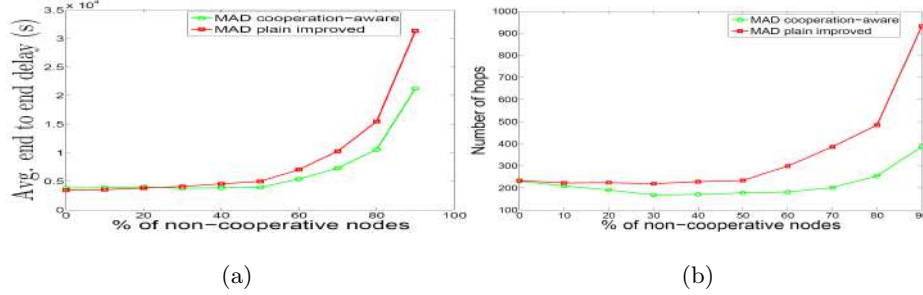


Fig. 3: Average (a) end-to-end delay and (b) number of hops vs non-cooperative environment (500 nodes and 120 km/h).

The results in Figs. 3a–3b provide direct evidence that both of the average end-to-end delay and the number of hops to delivery increase as the environment becomes more hostile. This happens for completely different reasons in the two versions of the protocol: The “plain” improved MAD takes the forwarding decision ignoring the notion of cooperation and the message may be “trapped” in non-cooperative nodes that handle its further routing inefficiently. On the other hand, cooperative-aware MAD encapsulates the notion of cooperation in the subjective density and thus the cooperative nodes are preferred as next hops.

## 5 On-Demand Beaconing: Periodic and Adaptive Policies for Effective Routing in Diverse Mobile Topologies

The already described contribution of the thesis, addresses issues regarded the context-aware routing policy including cooperation. Another important issue is the operational aspect of the signaling required for a realistic implementation of the routing protocol. In addressing this aspect, the thesis studies relevant on-demand beaconing techniques. A generic analysis is provided for the case of periodically issued beacons, linking the beacon period to the trade-off between the quality of neighborhood perception (determining the routing effectiveness) and the required amount of signaling (related to energy expenditure at the nodes). The analysis leads to upper and lower bounds for the length of the beacon period, expressed in terms of mobility characteristics [8].

For the balance between energy efficiency (and reduced beacon signaling) and the protocol efficiency, an empirical upper bound can be established by requiring that the fraction of nodes sensed is greater than or equal to the fraction of nodes missed, and from the analysis yields

$$T_b \leq 3.142r/V_c, \quad (4)$$

where  $T_b$  is the beacon period and  $V_c$  is the speed of the carrier node  $c$ .

On the other hand, it is not necessary to insist on very small values of  $T_b$ . Although the refresh of information is good for maintaining an updated status of these nodes, a too frequent such update is excessive, wastes energy and increases signaling. One can set an empirical lower bound, by requiring that the fraction of nodes repeatedly checked among those nodes sensed should not be greater than the complementary fraction of nodes checked once. The analysis leads to

$$T_b \geq 0.808r/V_c. \quad (5)$$

For a visual representation of the aforementioned concepts, Fig. 1b displays the fraction of missed nodes and nodes checked multiple times, as a function of  $\alpha$ .  $\alpha$  expresses  $T_b$  in multiples of the generic time constant  $T_r$ , which is the minimum time that must elapse before the area covered by the node at the beginning of the time duration is completely non-overlapping with the area covered at the end.

Also, the thesis investigates policies where the inter-beacon intervals vary adapting to the environment, an approach most beneficial when routing is based on metrics bearing some relevance to time. This is the case with the MAD routing protocol, which incorporates the notion of retaining time, and also a potential application for other instances of DREOT. Linking the beacon intervals to each time applicable retaining time leads to an effective and efficient beacon policy [8].

The performance of the proposed periodic and adaptive beaconing schemes is considered in the context of the MAD routing protocol. In the set of results to be presented in Fig. 4, we investigate the impact of parameter  $\alpha$  on the performance of the periodic beaconing scheme (the relevant results labeled ‘periodic’). At the same time, the results illustrate the relevance of the upper (4) and lower (5) bounds on parameter  $\alpha$ . Also, we compare the performance of the periodic with the adaptive beaconing scheme (the relevant results labeled ‘adaptive’). Each beaconing scheme is tested with two versions of MAD employing different expressions for the retaining time, the original simpler (1) (labeled ‘original MAD’) and the more accurate/refined (2) (labeled ‘improved MAD’). Mobile nodes move according to the random direction mobility model with zero pause time and constant speed. In order to compare the performance of the periodic and adaptive schemes, we present a topological scenario that corresponds to sparse density but high mobility, where the speed of the nodes is 50 km/h.

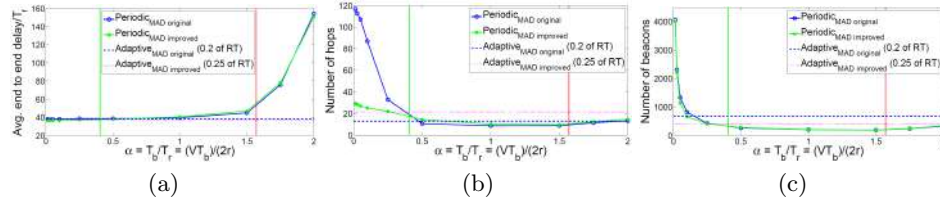


Fig. 4: Results for sparse and high mobility environments: (a) end-to-end delay; (b) number of hops; (c) total number of beacons.

By comparing the average end-to-end delay of the two periodic versions of MAD, the results indicate an increase as parameter  $\alpha$  increases. This is because a greater value of  $\alpha$  implies more neighbor nodes not detected as such from the current node. With respect to the average number of hops required for the message delivery from source to destination, the results show that the number of hops decreases with  $\alpha$ , until  $\alpha_{\text{upp}}$  is reached and then starts to increase. The initial decreasing trend is explainable if one recalls that higher values of  $\alpha$  correspond to routing in sparser environments. The increasing trend of the number of hops beyond some value of  $\alpha$  is due to the fact that overly long beacon periods result in the message being “lost” in the network. The number of beacons exhibits similar trends. Also, it can be said that the values of  $\alpha$  between  $\alpha_{\text{low}}$  and  $\alpha_{\text{upp}}$  constitute an appropriate zone for beacon period selection.

To compare the adaptive and periodic beaconing scheme, one starts from the figure for the end-to-end delay, finding the value of  $\alpha$  for periodic beaconing that matches the end-to-end delay obtained by the adaptive scheme. Then, one turns to the corresponding figure for the number of required beacons, and compares the number of beacons in the periodic scheme employing the said value of  $\alpha$  with the number of beacons required for the adaptive scheme. A similar procedure is involved when comparing in terms of the number of hops. By studying the figures, it can be seen that the adaptive scheme provides a better trade-off between performance and beacon messages than the periodic scheme.

## 6 Conclusions

In this PhD dissertation we provide methodologies for effective decision making in the fundamental networking functions in diverse network topologies, which is a cooperative problem solving. Towards contributing to the routing problem, the thesis introduces a routing protocol, called *Maximum Advance Decision* (MAD), based on the novel notion of the ‘*retaining time*’, which is capable of automatically adjusting for best performance under the conditions applicable each time. Along the process, the thesis develops techniques that are useful for studying more general times between events governing a decision-driven system, called ‘*Decision-Related Event Occurrence Time*’ (DREOT). Besides being useful on their own right, these results enable an in depth study of how the retaining time is affected by relevant network and nodal parameters and lead to an even greater improvement of the routing protocol. The ability of the retaining time to capture at the same time both routing environmental conditions and nodes’ characteristics, motivates an appropriate adjustment of the topological density to also reflect cooperation or other social aspects, through the novel notion of ‘*subjective density*’. Another important complementary issue is the operational aspect of the signaling required for a realistic implementation of the routing protocol. In addressing this aspect, the thesis proposes and examines two different ‘on-demand’ beaconing schemes, one with periodic beacons and the other with adaptively adjusted beacon intervals based on the notion of the retaining time. Finally, all these aspects are compared and verified through a rich set of numer-

ical and simulation results. The efficient and effective nature of the proposed techniques and notions reveals useful insights and provides guidelines as to how the decision policy of different operations is able to be self-adapted in diverse mobile environments.

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# Limitations of Linear Programming as a model of approximate computation

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**Abstract.** Linear programming has proved to be one of the most powerful and widely used tools in algorithm design and especially in the design of approximation algorithms. It has proved its expressive power by modeling diverse types of problems in planning, routing, scheduling, assignment, and design. However there are problems that seem to be very hard for linear programming. More specifically, the capacitated facility location problem (CFL) is an example of an important and well-studied problem for which, while it can be approximated within a constant factor using local search, it is not known to admit efficient relaxation based approximations.

In this thesis we take the direction of exploring the limitation of linear programming. Most of the thesis's results are concerned with linear programming approximability of the capacitated versions of the metric facility location problem such as the capacitated facility location (CFL). We give impossibility results in the hierarchy and in the extended formulations models and we also study another, independent family of relaxations which we call proper.

We show that the relaxations obtained from the natural LP at  $\Omega(n)$  levels of the semidefinite Lovász-Schrijver hierarchy for mixed programs, and of the Sherali-Adams hierarchy, the integrality gap is  $\Omega(n)$ , where  $n$  is the number of facilities. Our bounds are asymptotically tight. Then we prove that the standard CFL relaxation enriched with the submodular inequalities of [1] has also an  $\Omega(n)$  gap and thus not bounded by any constant. This disproves a long-standing conjecture of [24].

We propose a framework for proving lower bounds on the size of extended formulations. We do so by introducing specific types of extended relaxations that we call *product and distributional relaxations*. Then we show that for every approximate extended formulation of a polytope  $P$ , there is a product or distributional relaxation that has the same size and is at least as strong. We provide a methodology for proving lower bounds on the size of approximate product and distributional relaxations and, as an application of our method, we show for CFL an exponential lower bound on the size of a restricted type of extended formulations.

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# 1 Dissertation Summary

This thesis is naturally divided in three parts regarding the subfield in which the corresponding results reside: One part regarding the quality of solution for the CFL problem obtained by capitalizing on LP hierarchies, a second part, and perhaps the most important contribution of the thesis, regarding the development of a methodology for lower bounding the size of Extended Formulations and applications to the CFL problem, and one smaller part regarding the characterization of the strength of generalized configuration linear programs for CFL. In this overview we first define CFL and give a detailed background, and then we give an introduction to each topic corresponding to each part: we mention relevant work and we briefly present the exact contribution and results obtained by the research conducted in the context of this thesis.

## 1.1 Approximating Facility Location

Facility location is one of the most well-studied families of models in combinatorial optimization. In the *uncapacitated facility location* problem (UFL) we are given a set  $F$  of facilities and a set  $C$  of clients. We may open facility  $i$  by paying its opening cost  $f_i$  and we may assign client  $j$  to facility  $i$  by paying the connection cost  $c_{ij}$ . We are asked to open a subset  $F' \subseteq F$  of the facilities and assign each client to an open facility. The goal is to minimize the total opening and connection cost. The approximability of UFL was settled by an  $O(\log |C|)$ -approximation [15], which via a reduction from Set Cover is asymptotically best possible, unless  $P = NP$  [28]. In *metric* UFL the service costs satisfy the following variant of the triangle inequality:  $c_{ij} \leq c_{ij'} + c_{i'j'} + c_{i'j}$  for any  $i, i' \in F$  and  $j, j' \in C$ . This natural special case of UFL is approximable within a constant-factor, and many improved results have been published over the years. In those, LP-based methods, such as filtering, randomized rounding and the primal-dual method have been particularly prominent (see, e.g., [34]). After a long series of papers the currently best approximation ratio for metric UFL is 1.488 [25], while the best known lower bound is 1.463, unless  $P = NP$  ([33]).

CFL is the generalization of metric UFL where every facility  $i$  has a capacity  $u_i$  that specifies the maximum number of clients that may be assigned to  $i$ . In *uniform* CFL all facilities have the same capacity  $U$ . Finding an approximation algorithm for CFL that uses a linear programming lower bound was until recently a notorious open problem. The natural LP relaxations have an unbounded integrality gap and up to the recent breakthrough of [5], the only known  $O(1)$ -approximation algorithms were based on local search, with the currently best ratios being 5 [8] for the non-uniform and 3 [2] for the uniform case respectively. In the special case where all facility costs are equal, CFL admits an LP-based 5-approximation [24]. Williamson and Shmoys [34], stated the design of a relaxation-based algorithm for CFL as one of the top 10 open problems in approximation algorithms. Very recently, An et al. [5] gave a polynomial-time LP-based 288-approximation algorithm, thus answering the open question of [34]. The LP in [5] has exponential size and is not known to be separable in

polynomial time. Therefore the question on the existence of an efficient, compact, linear relaxation for CFL remains open. The series of our results regarding the LP-(in)approximability of CFL can be taken as very strong evidence that such a relaxation does not exist.

The Lower Bound Facility Location (LBFL) is in a sense the opposite problem to CFL. In an LBFL instance every facility  $i$  comes with a lower bound  $b_i$  which is the minimum number of clients that must be assigned to  $i$  if we open it. In *uniform* LBFL all the lower bounds have the same value  $B$ . LBFL is even less well-understood than CFL. The first approximation algorithm for the uniform case had a performance guarantee of 448 [32], which has been improved to 82.6 [3]. Both use local search. Interestingly, the LBFL algorithms from [32, 3] both use a CFL algorithm on a suitable instance as a subroutine.

## 1.2 Lift-and-Project methods and the resulting Hierarchies as model of LP computation

A lot of effort has been devoted to understanding the quality of relaxations of 0-1 polytopes obtained by an iterative lift-and-project procedure. Such procedures define hierarchies of successively stronger relaxations, where valid inequalities are added at each level. At level at most  $d$ , where  $d$  is the number of variables, all valid inequalities have been added and thus the integer polytope is expressed. Relevant methods include those developed by Balas et al. [7], Lovász and Schrijver [27] (for linear and semidefinite programs), Sherali and Adams [30], Lasserre [22] (for semidefinite programs). See [23] for a comparative discussion. The seminal work of Arora et al. [6], studied integrality gaps of families of relaxations for Vertex Cover, including relaxations in the Lovász-Schrijver (LS) hierarchy. This paper introduced the use of hierarchies as a restricted model of computation for obtaining LP-based hardness of approximation results.

We give impossibility results on arguably the most promising directions for obtaining efficient linear strengthened relaxations for CFL using hierarchies and, in doing so, we answer open problems from the literature.

Our first result of this part of our contribution is that there is an instance with  $\Theta(n)$  facilities and  $\Theta(n^4)$  clients on which the relaxations produced at  $\Omega(n)$  levels when the LS procedure is applied on the natural CFL LP have an integrality gap of  $\Omega(n)$ . The natural LP has a facility opening variable  $y_i$ , for every  $i \in F$ , and an assignment variable  $x_{ij}$ , for every  $i \in F$ , and client  $j \in C$ . We also extend the former result to the mixed  $LS_+$  hierarchy. This procedure is the stronger version of LS where one additionally requires that every protection matrix is positive semidefinite. The *mixed*  $LS_+$  procedure for a mixed integer program is the version of  $LS_+$  where one lifts only the 0-1 variables and requires that the resulting protection matrix is positive semidefinite (see, [7], [12]). We show that the  $\Omega(n)$  gap applies for  $\Omega(n)$  rounds of mixed  $LS_+$  as well.

We then show that the LPs obtained from the natural relaxation for CFL at  $\Omega(n)$  levels of the stronger  $SA$  hierarchy have a gap of  $\Omega(n)$  on the same family of instances used for the LS result, with  $|F| = \Theta(n)$  and  $|C| = \Theta(n^4)$ , giving the second contribution of this part. This result answers the questions of [26] and [4]

stated above as far as the natural LP is concerned. Our bound is asymptotically tight since the relaxation obtained at every level of the SA hierarchy is at least as strong as the one obtained at the same level of LS. We use a variation of the *local-to-global* method which was implicit in [6] for local-constraint relaxations and was then extended to the SA hierarchy in [13]. From a qualitative aspect, we give the first, to our knowledge, hierarchy bounds for a relaxation where variables have more than one type of semantics, namely the facility opening and the client assignment type. Compare this, for example, with the Knapsack and Max Cut LPs that contain each one type of variable.

Our third contribution in this part is that the *submodular* inequalities introduced in [1] for CFL fail to reduce the gap of the classic relaxation to constant. These constraints generalize the flow-cover inequalities for CFL. Thus we disprove the long-standing conjecture of [24] that the addition of the latter to the classic LP suffices for a constant integrality gap. Although this is not a result that concerns linear programming hierarchies, we included its presentation in that part of the thesis because the methodology we use is inspired by the local-to-global method and thus our proof deviates from standard integrality gap constructions. In fact we take the idea of fooling local constraints a little further: the bad solution fools every inequality  $\pi$  because its part that is *visible* to  $\pi$ , i.e., the variables in the support of  $\pi$ , can be extended to a solution that is a convex combination of feasible integer solutions for that instance or it is a convex combination of feasible solutions to another instance for which the same inequality is valid. Our proof relies on simple structural properties of the inequalities, disregarding the exact coefficients of the variables.

### 1.3 Extended Formulations: the currently most general mode of LP computation

In the past few years there has been an increasing interest in exposing the limitations of compact LP formulations for combinatorial optimization problems. The goal is to show a lower bound on the size of *extended formulations (EFs)* for a particular problem. Extended formulations add extra variables to the natural problem space; the increase in dimension may yield a smaller number of facets. The minimum size over all extended formulations is the *extension complexity* of the corresponding polytope. A superpolynomial lower bound on the extension complexity is of intrinsic interest in both polyhedral combinatorics and combinatorial optimization and implies that there is no polynomial-time algorithm relying purely on the solution of a compact linear program.

In the seminal paper of Yannakakis [35] the problem of lower bounding the size of extended formulations was considered for the first time: exponential lower bounds were proved for symmetric extended formulations of the matching and TSP polytopes. Yannakakis [35] identified also a crucial combinatorial parameter, the *nonnegative rank* of the slack matrix of the underlying polytope  $P$ , and he showed that it equals the extension complexity of  $P$ . A strong connection of the extension complexity of a polytope to communication complexity was made in [35], by showing that the nonnegative rank of the slack matrix is at least

the size of its minimum rectangle cover. That connection has been exploited in several results on the extension complexity of polytopes.

Fiorini et al. [14] lifted the symmetry condition on the result of [35] regarding the TSP polytope, giving the first example of a polytope with exponential extension complexity and thus answering a long-standing open problem of [35]. Recently, Rothvoß [29] removed the symmetry condition for the matching polytope as well, answering the second long-standing open question of [35]. This was done by a breakthrough in bounding a refined version of the rectangle covering number.

A more general question is that of the size of approximate extended formulations. This problem was first considered in [10] where the methodology of [14] was extended to approximate formulations and an exponential bound for the linear encoding of the  $n^{1/2-\varepsilon}$ -approximate clique problem was given.

In [11] it was proved that in terms of approximating maximum constraint satisfaction problems (CSPs), LPs of size  $O(n^k)$  are exactly as powerful as  $O(k)$ -level relaxations in the Sherali-Adams hierarchy. Their proof differs from previous work in showing that polynomials of low degree can approximate the functional version of the factorization theorem of [35].

**Our contribution on Extended Formulations** In the relevant part of the thesis we propose a new intuitive, geometric approach for proving lower bounds on the size of approximate extended formulations that relies on an insight on the expressive strength of “strong” sets of variables and encodings. Our contribution is summarized by the following.

First we introduce two very strong families of extended formulations (or relaxations) of a given polytope which we call *product formulations* and *distributional formulations*. The product relaxations are inspired by the study of the Sherali-Adams hierarchy – the variables have the intuitive meaning of corresponding to products over sets of variables from the original space. The distributional are intended to encode the problem in such a way that the feasible region is a straightforward distribution of (convex combination of) feasible integer solutions. (See Section 2 for the necessary definitions).

We prove in Theorem 1 that for any  $\rho$ -approximate extended formulation of a 0-1 polytope there is a product (distributional) formulation of the same size that is at least as strong. Theorem 1 reduces lower bounding the size of an extended formulation, which uses some unknown space and encoding, of a polytope  $P$ , to lower bounding the size of product (distributional) formulations of  $P$ . In the product (distributional) space we have the concrete advantage of knowing the section of the target relaxation. We extend the definition of product relaxations and our methodology to mixed integer sets. However in this case we are able to show that *mixed product relaxations* are at least as powerful as a special family of extended formulations.

Then we propose a methodology for proving lower bounds for relaxations for which the encoding of solutions is known, and in particular for product (distributional) formulations. The method is the following: first we define a set

of vectors in the space of the relaxation such that for each one of those vectors there is an admissible objective function witnessing an integrality gap of  $\rho$ . We call that set of vectors the *core*. Then we show that, for any partition of the core into fewer than  $\kappa$  parts, there must be some part containing a set of conflicting vectors. A set of infeasible vectors is *conflicting* if its convex hull has nonempty intersection with the convex hull of  $\{z^x \mid x \in P(x) \cap \{0, 1\}^n\}$ , which is always included in the feasible region of a product relaxation – here  $z^x$  is the encoding of feasible solution  $x$  to the variables of product formulations. Thus, we get that at least  $\kappa$  inequalities are needed to separate the members of the core from the feasible region and so  $\kappa$  is a lower bound on the size of any  $\rho$ -approximate product formulation. By considering the hypergraph whose set of vertices corresponds to the aforementioned set of vectors and whose set of hyperedges corresponds to the sets of conflicting vectors, the chromatic number of the hypergraph is a lower bound on the size of every  $\rho$ -approximate extended formulation. Moreover, there is always a core such that the chromatic number of the resulting, possibly infinite, hypergraph equals the extension complexity of the polytope at hand. Thus we give a characterization of extension complexity which can be seen as an alternative to the nonnegative rank of the slack matrix.

We exhibit a concrete application of our methodology by proving an exponential lower bound on the size of any  $O(N)$ -approximate mixed product relaxation for the CFL polytope, where  $N$  is the number of facilities in the instance. This result can be shown to imply that the  $\Omega(N)$ -level SA relaxation for CFL, which is obtained from any starting LP of size  $2^{o(N)}$  defined on the classic set of variables, has unbounded gap  $\Omega(N)$ . This settles the open question of [4] whether there are LP relaxations upon which the application of lift-and-project methods captures the strength of preprocessing steps for CFL. This result establishes for the first time such a trade-off for a SA procedure that is independent of the starting relaxation  $K$ .

#### 1.4 The strength of generalized configuration linear programs for capacitated versions of the facility location problem

In the relevant part of this thesis we introduce and study the family of proper relaxations which are configuration-like linear programs. The so-called *Configuration LP* was used by Bansal and Sviridenko [9] for the Santa Claus problem and has yielded valuable insights, mostly for resource allocation and scheduling problems (e.g., [31]). The analogue of the Configuration LP for facility location already exists, it is the *star relaxation* (see, e.g., [16]). In a star relaxation every variable corresponds to a *star*, i.e., a facility  $f$  and a set of clients assigned to  $f$ . The natural star relaxation for CFL and LBFL is equivalent to the standard LPs so it has an unbounded integrality gap. We generalize the idea of a star by introducing what we call *classes*. A *class* consists of a set with an arbitrary number of facilities and clients together with an assignment of each client to a facility in the set. The definition of a class can thus vary from simple, “local” assignments of clients to a single facility, to “global” snapshots of the instance that express the assignment of clients to a large set of facilities. A *proper relaxation*

for an instance is defined by a collection  $\mathcal{C}$  of classes and a decision variable for every class. We allow great freedom in defining  $\mathcal{C}$ ; the only requirement is that the resulting formulation is symmetric and valid. The *complexity*  $\alpha$  of a proper relaxation is the maximum fraction of the available facilities that are contained in a class of  $\mathcal{C}$ . Proper LPs are stronger than the standard relaxation. One can easily construct infinite families of instances where, by increasing the complexity in a proper relaxation, one cuts off more and more fractional solutions. We characterize the behavior of proper relaxations for CFL and LBFL through a sharp threshold result: anything less than maximum complexity results in a gap that is not bounded by any constant, while there are proper relaxations of maximum complexity with a gap of 1.

### 1.5 Publications

The publications that resulted from the work presented in this thesis include the following: [20] contains the results on the SA hierarchy, the flow-cover inequalities and the proper relaxations. The results regarding the LS hierarchy combined with the results of [19] and [20] were published in [21]. The results regarding the Extended Formulations are contained in [18].

## 2 Preliminaries on Extended Formulations

Given a polyhedron  $K(x, y) = \{(x, y) \in \mathbb{R}^d \times \mathbb{R}^{d_y} \mid Ax + By \leq b\}$  the *projection to the  $x$ -space* is defined as  $\{x \in \mathbb{R}^d \mid \exists y \in \mathbb{R}^{d_y} : Ax + By \leq b\}$ , denoted as  $\text{proj}_x(K(x, y))$ . An *extended formulation* of a polyhedron  $P(x) \subseteq \mathbb{R}^d$  is a linear system  $K(x, y) = \{(x, y) \in \mathbb{R}^d \times \mathbb{R}^{d_y} \mid Ax + By \leq b\}$  such that  $\text{proj}_x(K(x, y)) = P(x)$ . The *size* of a polyhedron  $P(x)$  is the minimum number of inequalities in its halfspace description. The *extension complexity* of  $P(x)$  is the minimum size of an extended formulation of  $P(x)$ .

We define now  $\rho$ -approximate formulations as in [10]. Given a combinatorial optimization problem  $T(S, f)$ , a *linear encoding* of  $T$  is a pair  $(L, O)$  where  $L \subseteq \{0, 1\}^*$  is the set of encodings of *feasible solutions* to the problem and  $O \subset \mathbb{R}^*$  is the set of encodings of the *admissible objective functions*. An instance of the linear encoding is a pair  $(d, w)$  where  $d$  is a positive integer defining the dimension of the instance and  $w \subseteq O \cap \mathbb{R}^d$  is the set of admissible cost functions for instances of dimension  $d$ . Solving the instance  $(d, w)$  means finding  $x \in L \cap \{0, 1\}^d$  such that  $w^T x$  is either maximum or minimum, according to the type of problem  $T$ . Let  $P = \text{conv}(\{x \in \{0, 1\}^d \mid x \in L\})$  be the corresponding 0-1 polytope of dimension  $d$ . Given a linear encoding  $(L, O)$  of a maximization problem, the corresponding polytope  $P$ , and  $\rho \geq 1$ , a  $\rho$ -approximate extended formulation of  $P$  is an extended relaxation  $Ax + By \leq b$  of  $P$  with  $x \in \mathbb{R}^d, y \in \mathbb{R}^{d_y}$  such that

$$\begin{aligned} \max\{w^T x \mid Ax + By \leq b\} &\geq \max\{w^T x \mid x \in P\} && \text{for all } w \in \mathbb{R}^d \text{ and} \\ \max\{w^T x \mid Ax + By \leq b\} &\leq \rho \max\{w^T x \mid x \in P\} && \text{for all } w \in O \cap \mathbb{R}^d. \end{aligned}$$

For a minimization problem, we require

$$\begin{aligned} \min\{w^T x \mid Ax + By \leq b\} &\leq \min\{w^T x \mid x \in P\} && \text{for all } w \in \mathbb{R}^d \text{ and} \\ \min\{w^T x \mid Ax + By \leq b\} &\geq \rho^{-1} \min\{w^T x \mid x \in P\} && \text{for all } w \in O \cap \mathbb{R}^d. \end{aligned}$$

The  $\rho$ -approximate extension complexity of 0-1 integer polytope  $P(x) \subseteq [0, 1]^d$  is the minimum size of a  $\rho$ -approximate extended formulation of  $P$ .

We turn now to define a generic extended formulations that will play a central role.

**Definition 1.** Given a 0-1 integer polytope  $P(x) \subseteq [0, 1]^d$ , a product formulation  $D(z)$  of  $P(x)$  is an extended formulation  $D(z)$  of  $P(x)$ , where  $z \in \mathbb{R}^{2^d - 1}$  and for every nonempty subset  $\mathcal{E} \subseteq \{x_1, x_2, \dots, x_d\}$  of the original variables, we have a variable  $z_{\mathcal{E}}$ , (where  $z_{\{x_i\}}$  denotes  $x_i$ ,  $i = 1, \dots, d$ ). For any feasible integer solution  $x^s \in P(x) \cap \{0, 1\}^d$  the vector  $z^s$ , whose components are defined as  $z_{\mathcal{E}}^s = 1$  iff all variables in  $\mathcal{E}$  have value 1 in  $x^s$  and  $z_{\mathcal{E}}^s = 0$  otherwise, is feasible for any product formulation  $D(z)$  of  $P(x)$ . We will refer to  $z^s$  as the encoding of the feasible integer solution  $x^s$  in the product variables.

Note that the lifted polytope obtained from some specific linear relaxation of the 0-1 polytope  $P(x)$ , at any level of the SA hierarchy, after linearization and before projection to the original variables, is a (mixed) product relaxation.

### 3 The expressive power of product relaxations

In this section we show the following. For every 0-1 polytope  $P(x)$  and every (approximate) extended formulation  $Q(x, y) = \{(x, y) \in \mathbb{R}^{d_x} \times \mathbb{R}^{d_y} \mid Ax + By \leq b\}$  of  $P(x)$  there is a product formulation  $T_Q$  which has the size of  $Q(x, y)$  and is at least as strong in terms of approximability. Similarly, we show that there is a distributional formulation  $R_Q$  of the same as  $Q(x, y)$  and at least as strong.

A substitution  $T$  for the product space is a linear map of the form  $y = Tz$  where  $T$  is a  $d_y \times (2^{d_x} - 1)$  matrix and  $z$  is a  $2^{d_x} - 1$  dimensional vector having a coordinate  $z_{\mathcal{E}}$  for each nonempty set  $\mathcal{E}$  of the form  $\{x_i \mid i \in S \subseteq \{1, \dots, d_x\}\}$ . For any substitution  $T$ , the translation of  $Q(x, y)$ , denoted  $T_Q$ , the formulation resulting by substituting  $T_{(i)}z$ , for  $y_i$ ,  $i = 1, \dots, d_y$ . Here  $T_{(i)}$  denotes the  $i$ th row of  $T$ . We require that  $T_Q$  is a product formulation (see Definition 1) and we say that we have a translation of  $Q$  to product formulations (recall that the original variables  $x_i$  coincide with the variables  $z_{\{x_i\}}$ ). Observe that the number of inequalities of  $T_Q$  is the same as in  $Q(x, y)$ . The translation may heighten exponentially the dimension, but, since our methodology will give lower bounds on the size of the product formulations, those bounds apply to the size of  $Q(x, y)$  as well. A substitution  $T$  for the distributional space and a translation to distributional formulations is defined similarly.

**Theorem 1.** Given a 0-1 polytope  $P(x) \subseteq [0, 1]^{d_x}$ , for every polytope  $Q(x, y)$  such that  $P(x) \subseteq \text{proj}_x(Q(x, y))$  there is a translation  $T_Q$  to product formulations such that  $P(x) \subseteq \text{proj}_x(T_Q) \subseteq \text{proj}_x(Q(x, y))$ .



*Proof.* We shall give a substitution  $T$  for the variables  $y \in \mathbb{R}^{d_y}$  of  $Q(x, y)$  so that the theorem holds. Let  $g(x)$  be a section of  $Q(x, y)$  (recall that a section associates every feasible 0-1 vector  $x$  of  $P(x)$  to a specific  $y$  such that  $(x, y) \in Q(x, y)$ ). We denote by  $(p, 1) \in \mathbb{R}^{n+1}$  the vector resulting from  $p \in \mathbb{R}^n$  by appending the scalar 1 as an extra coordinate.

Observe that a product variable  $z_{\mathcal{E}}$  behaves, as far as the encodings  $z^s$  of solutions  $x^s \in P(x) \cap \{0, 1\}^{d_x}$  to product variables are concerned, like the monomial  $\prod_{x_i \in \mathcal{E}} x_i$  would. Those monomials plus the constant 1 form the *Fourier basis*. Likewise we can see a variable  $y_i$ , as far as the encodings  $x^s, y^s$  of solutions  $x^s \in P(x) \cap \{0, 1\}^{d_x}$  are concerned, as a boolean function  $y_i(x) : \{0, 1\}^{d_x} \rightarrow \mathbb{R}$  such that  $y_i(x^s) = y_i^s$ . By basic functional analysis (see, e.g., [17]), we have that every boolean function  $y_i(x)$  has a unique Fourier representation  $y_i(x) = \sum_{\mathcal{E} \subseteq \{x_i | i=1, \dots, d_x\}} a_{\mathcal{E}}^{y_i} \prod_{x_i \in \mathcal{E}} x_i$ . The intuition is that we will use the encodings  $z^s$  to product variables to simulate the encodings  $y^s$ . So we define the substitution  $T_i$  for a variable  $y_i$  as follows:

$$y_i = \sum_{\mathcal{E} \subseteq \{x_i | i=1, \dots, d_x\}} a_{\mathcal{E}}^{y_i} z_{\mathcal{E}} \quad (1)$$

In the above expression we assume, for notational convenience that,  $z_{\emptyset} = 1$ . Recall that product variables are defined for nonempty sets.

Obviously  $\text{proj}_x(T_Q) \subseteq \text{proj}_x(Q(x, y))$ : from any feasible solution  $(x^0, z^0)$  of  $T_Q$  we can derive a feasible solution  $(x^0, y^0)$  of  $Q(x, y)$  by setting  $y^0$  equal to  $Tz^0$ .

We will now show that  $P(x) \subseteq \text{proj}_x(T_Q)$  or, more specifically, that the encodings  $z^s$  of solutions to product variables are feasible for  $T_Q$  as required by the definition of product relaxations. Observe that by letting the  $z$  vector take the value  $z^s$  for some  $s \in P \cap \{0, 1\}^{d_x}$ , by (1) we get that the quantities involved in the inequalities of  $T_Q$  are the exact same quantities involved in the corresponding inequalities of  $Q(x, y)$  for  $(x, y) = (x^s, y^s)$ . By definition  $(x^s, y^s)$  is feasible for  $Q(x, y)$  and thus  $z^s$  is feasible for  $T_Q$ .

**Corollary 1.** *A lower bound  $b$  on the size of any product relaxation  $D$  which is a  $\rho$ -approximate extended formulation of the 0-1 polytope  $P(x)$ , for  $\rho \geq 1$ , implies a lower bound  $b$  on the size of any  $\rho$ -approximate extended formulation  $Q(x, y)$  of  $P(x)$ .*

## 4 Conclusion

In the context of this thesis we exposed the limitations of linear programming methods for providing satisfactory approximations to assignments problem with restrictions such as capacities. In particular we showed that the unboundedness of the integrality gap of CFL or LBFL relaxations persists even after applying the tightenings of the LS and SA hierarchies. We did so by proving the feasibility of a bad fractional solution for an asymptotically tight number of levels. We also

proved that the submodular inequalities do not reduce the integrality gap to constant. Then, while turning our attention to the more general model of extended formulations, we devised a methodology for lower bounding the extension complexity which also serves as a characterization of the extension complexity. We applied our method to derive tight bounds on the size of mixed product relaxations which result also implies tight SA gaps regardless of the initial relaxation. Lastly, we proved similar negative results for families of proper relaxations that capture general configuration LPs. The obtained results answered a number of interesting open questions and conjectures from the relevant literature.

In the recent work of An et al. [5] the first constant factor LP-based approximation algorithm for CFL was given. However, the proposed relaxation is exponential in size and, according to the authors, it is not known to be separable in polynomial time. A natural question that arises is whether there is a polynomially-sized relaxation achieving a constant integrality gap. An interesting direction is that of determining the minimum size of an approximate extended formulation of the CFL polytope, which our results arguably suggest to be exponential. We leave this as an open problem.

Regarding our lower bounding methodology for extended formulations, the proof of our result for mixed product relaxations for CFL made use of a core whose underlying hypergraph is actually a graph and moreover a clique. To generalize this result to product formulations or distributional formulations, or to prove bounds on the extension complexity of other polytopes, we believe that the power of general hypergraphs needs to be exploited. Observe that our methodology requires only the existence of a suitable core, and thus, one could possibly employ probabilistic arguments to prove the existence of suitable hypergraphs of high chromatic number.

In the case of mixed integer polytopes, we leave as an open problem whether the mixed product relaxations are strong enough to simulate any extended formulation, as is the case for product relaxations and 0-1 polytopes.

We also believe that it would be interesting to revisit polytopes, whose extension complexity has been shown to be large, and provide independent proofs using our method, ideally by improving on the known bounds. Moreover, as we showed for CFL using product or distributional formulations one can provide lower bounds as well and this can be of help in settling the extension complexity.

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# Selfish Behavior and Compact Representation in Routing and Information Networks

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**Abstract.** In this short note we summarize our results on the study of selfish user behavior in modern networks and the compact representation of such networks in systems' level. In the first part of the dissertation we use principles of Game Theory to build frameworks for the theoretical study of problems arising in routing networks and the worldwide web due to the selfish nature of their users, while in the second one we move to the important technical issue of representing such networks so that they can fit in the computer's main memory but also boost the efficiency of critical applications that run over these networks. The results include the introduction and study of the class of congestion games with time-dependent strategies, the analysis of game-theoretic aspects of link placement in the worldwide web and the design and evaluation of a network compression algorithm that outperforms the state-of-the-art method by achieving a better compression ratio and retrieval time of the network's elements.

## 1 Introduction

During the last fifteen years, the explosive growth of the internet and its use in everyday life has given rise to various types of networks created by human activity, without any central design or control. The most well known example is the worldwide web (WWW), which became an object of study when it started being used by a huge number of users and thus attracted a lot of information-oriented and economic activity. More recently a number of other networks available through internet have been developed, that also offer abstractions of the social context of the users, referred to as social networks. The development of the internet has also motivated the study of routing networks, as there are usually various alternates for the communication between two network entities.

The popularity of such networks has clearly affected the incentives of their users. For instance, web page authors are interested in establishing hyperlinks that increase not only the quality but also the *popularity* of their pages. However, attracting users is not desirable in all types of networks. For example, in routing networks it is usually in users' interest to *avoid* crowded communication

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channels. Apparently, since we need to be able to understand the mechanisms behind the creation of such networks, network analysis has become an intriguing object of study.

Apart from the user point of view consequences of the networks' popularity, there are also consequences from the system point of view. Perhaps the most basic such issue is that the network representations cannot fit in a computer's main memory, leading to two drawbacks: a restriction on the network size that can be handled, or poor performance of the critical applications that run over such network infrastructures, implied by the usage of secondary memory. Neither of them is acceptable, so we need compressed representations that allow fast access to the network's elements.

The main questions that motivated our research are the following:

- What is the optimal *timing* for a user to enter a *routing network*?
- What is the optimal *linking* strategy in the *worldwide web*?
- How can we efficiently *represent* the WWW, social or routing networks?

This dissertation has been conducted along the two directions sketched above. The first one is the study of *selfish behavior* in various networks of special interest in our days, in particular routing networks and information networks. In both types the users are trying to maximize their own payoff from the network; the different incentives, however, make the individual study of each network type necessary. In the case of routing networks, which we study theoretically using congestion games, we introduce and study the class of congestion games with time-dependent strategies. We model the routing networks using congestion games, because it is considered the most appropriate approach for the theoretical study of congestion networks. In a more practical level, we model and study the worldwide web, which is the largest information network in our days, from a game-theoretic point of view that captures naturally the users' incentives. The second direction has to do with an important technical issue, the representation of these network structures and more specifically with their *compact-yet-efficient* representation, that can boost the performance of critical applications that run on such networks. We study the common properties of the networks of interest and exploit them to design an efficient lossless compression method.

The contribution of the thesis [35] is threefold, and we outline it in the following sections. We model the selfish behavior of the users of such systems using the Game Theory framework. Regarding the class of congestion games with time-dependent strategies, we describe its structural properties and compute its Nash equilibria and prices of anarchy and stability. A brief illustration of these results can be found in section 2.1. Then, in section 2.2, we give an idea of our analysis of the game-theoretic aspects of the worldwide web, considering a link placement game among the web authors and computing their exact and approximate best responses. Both these objects are frameworks for studying classes of problems. Finally we focus on the representation of networks created by human activity, with routing networks, social networks and the worldwide web being the most popular examples. We propose a network representation method

that outperforms the state-of-the-art method in terms of compressed size and access time to the elements of the network. Section 3 outlines our approach, its analysis and experimental evaluation.

Most of these results have appeared in [24,25,26]. What follows is a brief presentation of the research topics and results of the thesis, avoiding technical details.

## 2 Selfish Behavior in Routing and Information Networks

We first consider routing (or traffic) networks and study the effect of the participation timing in such systems (Section 2.1). We then consider information networks, focus on WWW and highlight its game-theoretic aspects (Section 2.2).

### 2.1 Congestion games with time-dependent strategies

In the last dozen years, the concepts of the price of anarchy (PoA) and stability (PoS) have been successfully applied to many classes of games, most notably to *congestion* games and its relatives [23,40,30]. In congestion games, the players compete for a set of resources, such as facilities or links; the cost of each player depends on the number of players using the same resources; the assumption is that each resource can be shared among the players, but with a cost. Another interesting class of games are the *contention* games [16] in which the players again compete for resources, but the resources cannot be shared. If more than one players attempt to share a resource at the same time, the resource becomes unavailable and the players have to try again later. There are however interesting games that lie *between* the two extreme cases of the congestion and contention games. For example, the game that users play for dealing with congestion on a network seems to lie in between—the TCP congestion control policy is a strategy of this game. Timing is part of the strategy of the players (as in contention games) and the latency of a path depends on how many players use its edges (as in congestion games).

In our work, we attempted to abstract away the essential features of these games, to model them, and to study their properties, their Nash equilibria, and their price of anarchy and stability. The games that we consider are essentially congestion games with the addition of time dimension. The difference with congestion games is that players now don't simply select which path to use, but they also decide *when* to enter the system, and hence the delay they suffer depends on the number of users that use *the same path edges at the same time* as they do.

*Related work.* Contention resolution in communication networks is a problem that has attracted the interest of diverse communities of Computer Science, due to the fact that contention is inherent in many critical network applications. One of them is the design of multiple access protocols for communication networks, such as Slotted Aloha. In such protocols, the increase of users of the

network incurs a large number of collisions and subsequently poor utilization of the system's resources.

During the last four decades many more refined multiple access protocols have been proposed to increase the efficiency of Aloha, the vast majority of which assume that the agents follow the protocol, even if they might prefer not doing so. Recently, slotted Aloha has been studied from a game-theoretic point of view, trying to capture the selfish nature of its users [2,3,27,16,15].

Routing in networks by selfish agents is another area that has been extensively studied based on the notion of the price of anarchy (PoA) [23] and the price of stability (PoS) [4]. The PoA and the PoS compare the social cost of the worst-case and best-case equilibrium to the social optimum. Selfish routing is naturally modeled as a congestion game. The class of congestion or potential games [38,29] consists of the games where the cost of each player depends on the resources he uses and the number of players using each resource.

All these models share with our work the interest in game-theoretic issues of timing in routing, but they differ in an essential ingredient: in our games, timing is the most important part of the players strategy, while in the previous work, time delays exist because of the interaction of the players; in particular, *in all these models the strategy of the players is to select only a path*, while in our games the strategy is essentially the timing.

*Our contribution.* We introduce and study the class of congestion games with time-dependent strategies. Consider a link or facility  $e$  of a congestion game with latency function  $\ell_e$ . In the congestion game the latency that a player experiences on the link is  $\ell_e(k)$ , where  $k$  is the number of players that use the link. In our model however, since the players may also decide when to start, we have to redefine the latency.

We propose the following latency models for the links:

**The boat model**, in which only the group of players that start together affect the latency of the group. Imagine that one boat departs from the source of the link at every time step; all players that decide to start at time  $t$  enter the boat which takes them to their destination; the speed of the boat depends only on the number of players in the boat and it is independent of the players on the other boats.

**The conveyor belt model**, in which the latency of a player depends on the number of other players using the link at the same time, regardless if they started earlier or later. Specifically, the link resembles a conveyor belt from the source to the destination; the speed of the belt at every time depends on the number of people on it.

We consider only symmetric strategies based on the assumption that these games are played by many players with no coordination among them. Moreover we assume non-adaptive strategies, in which the players decide on their strategy before the game starts. *A pure strategy of a player consists of a path and a starting time.*



We first study the structural properties of the boat and conveyor belt games. We prove that the boat games are congestion games; in contrast, we give examples which show that conveyor belt models are not in general congestion games with the exception of the case of two players. In fact, even simple games with 3 players may not even possess pure Nash equilibria.

We characterize the symmetric Nash equilibria of the boat model game for parallel links of affine latency functions, i.e.,  $\ell_e(k) = a_e k + b_e$ , and any number of players. We show that there is a unique symmetric mixed Nash equilibrium for these games. At the Nash equilibrium the probability that a player starts at time  $t$  drops linearly on  $t$ .

We also compute the optimal symmetric solution. Interestingly, in both the boat and conveyor belt model, the optimal symmetric strategy has exactly the same form with the Nash equilibria but it is less aggressive. That is, in the optimal symmetric strategy the probabilities drop also linearly in time but they are spread out to more strategies. In particular, the optimal strategy is a Nash equilibrium of a game with higher latency functions (by almost a factor of 2). A similar bicriteria relation between the Nash equilibria and the optimal solution has been observed in simple congestion games before [40].

From the characterization of the Nash equilibria and the optimal strategy, we get that the price of anarchy and stability is very low  $3\sqrt{2}/4 \approx 1.06$ . This is the price of anarchy (and stability) when we fix the latencies and let the number of players tend to infinity; when the latency function is tailored to the number of players  $n$ , the price of anarchy can be as high as  $8n/(7n + 1)$ .

We also study the class of conveyor belt games. These are more complicated games and here we consider only two players and arbitrary latency functions; for two players the class of affine and the class of arbitrary latency functions are identical. We again characterize the Nash equilibria, the optimal solution, and we compute the PoA and the PoS. Specifically, we show that there exists a unique symmetric Nash equilibrium for the conveyor belt model in which the players assign non-zero probabilities to multiples of  $\ell_e(1)$  and these probabilities drop linearly. The explanation of the nature of these equilibria is the following: a player attempts with some probability to start at time  $t = 0$ ; the probability has to balance the risk of the other player starting also at time  $t = 0$  and the delay incurred by waiting. The interesting property of the Nash equilibrium is that the player waits enough time steps in order to avoid interference with the other player, given that he had started at time  $t = 0$ . After exactly  $\ell_e(1)$  steps, with some probability, the player attempts again and the process is repeated.

The price of anarchy and stability is (for large latencies) again approximately  $3\sqrt{2}/4 \approx 1.06$ . This is the price of anarchy we computed for the boat model, but the relation is not as straightforward as it may appear: in the boat model we take the limit as the number of players tends to infinity, while in the conveyor model, we take the limit as the latencies tend to infinity. In fact, the latter is the same limit as keeping the latencies steady and letting the time step to tend to 0 (thus approximating a continuous-time protocol).

To our knowledge, these games differ significantly from the classes of congestion games that have been studied before. Also, the techniques developed for bounding the PoA and the PoS for congestion games do not seem to be applicable in our setting. In particular, the smoothness analysis arguments [13,39,14] do not seem to apply because we consider symmetric equilibria. In fact, the focus and difficulty of our analysis is to characterize the Nash equilibria and not to bound the PoA (or PoS).

The results of this work are detailed in chapter 3 of the thesis [35] and have been published in [24]. In the context of our ongoing research on this problem, we have extended our results to more general network models.

## 2.2 Game-theoretic aspects of the Worldwide Web

The worldwide web has been the focus of an enormous amount of research in the last fifteen years and several models have been proposed for it. These models aim at our understanding of the properties and evolution of the web, and assist us in designing more efficient web algorithms and applications, for example search engines. Recently, the exploitation of web's link structure by the search engines as well as the emergence of advertising links have given new incentives to link placement: strategic web page owners now explicitly attempt to boost their reputation and monetary revenue by careful selection of links, and Search Engine Optimization (SEO) has grown into a billion-dollar industry. Therefore Game Theory seems to provide the appropriate framework for studying the evolution of the web. Moreover, the impact of advertising links on the link structure of the web, and consequently on the relative importance of web pages, is unknown.

In our work we introduce a game-theoretic model for the worldwide web that captures the selfish nature of web page authors. In our model the page authors decide which advertising links to buy in order to maximize their revenue, which depends on the traffic their page attracts. We use Google PageRank as a measure of traffic. Based on this model, we study the game-theoretic aspects of the worldwide web.

*Related work.* The first attempts to model the web graph coincide temporally with the development of successful web search algorithms which were based, partially, on the link structure of the web, with PageRank [34] and HITS [21] being the most well-known examples.

Since then many models have been proposed for the web graph, which try to predict its structural properties. Classifying the models according to the deemed linking incentive, we get the following classes: *random graph models*, in which new nodes link to existing ones with high degree or PageRank; *economic models*, in which the nodes endorse existing ones that are regarded as good web search results; and *game theoretic models*, in which nodes explicitly try to maximize their own PageRank and/or revenue.

The decisions on the link structure of each web page are made locally, with each page owner trying to maximize the value and importance of her own page.

Therefore game theory appears as the proper framework for modeling the link establishment process, and we can think of the web as the equilibrium of some network creation game among its users. During the last eight years, several aspects of linking in the web have been studied from a game theoretic point of view. The inner structure of a web site with strategic owner is studied in [18]. All other game theoretic models for the web concern the establishment of external links; either *reference only* [17,11] or *reference and advertising* [20,22].

*Our contribution.* We introduce and study a model for the web graph, in which selfish page owners aim at maximizing their PageRank and revenue by purchasing the appropriate incoming links to their page. We assume two different approaches for link pricing: fixed-prices and prices-per-click.

Our model is a game played among the web page owners, in which each one aims at purchasing the set of new incoming links to her page that maximizes the revenue from the page, i.e., the page's PageRank minus the prices of these links. We call this game the *web game*. We show that the web game is not a potential game for three or more players. The problem of finding the best response for each player given the strategies of the rest players is formulated as follows: Given a directed graph  $G = (V, E)$ , a node  $u \in V$  and the price  $p_i$  of the links emanating from node  $i$  for all  $i \in V$ , compute the set of new backlinks of  $u$  that maximize  $\pi_u - \sum_{j:j \rightarrow u} p_j$ , where  $\pi$  is the PageRank vector. We show that the best response computation is *NP*-hard, since the LINK BUILDING problem, which is known to be *NP*-hard (in fact it has no FPTAS unless  $P = NP$ ) [31], reduces to it. It follows that verifying a Nash equilibrium is *NP*-hard as well, since we have to verify that each player plays her best response given the strategies of the rest players, so we are interested in approximate best responses. In [33,32] the authors propose a constant factor approximation algorithm for the link building problem. We employ it to compute an *approximate best response* for the prices-per-click model in our web game.

The results of this work are presented in detail in chapter 4 of the thesis [35].

### 3 Compact Representation of Routing and Information Networks

Real-world systems and phenomena that involve interactions among various entities are being modeled using *graphs* for decades now. The recent explosive growth of large-scale systems that are traditionally modeled as graphs, the worldwide web and social networks being typical examples, has intensified the need for compact-yet-efficient representations of graphs. In particular, we need compressed graph representations that allow *mining* without decompressing the whole graph. In this way, algorithms and applications with tasks that correspond to graph mining problems, can take advantage of such representations to boost their performance, as they can run in main memory over much larger graphs using their compressed representations instead of the plain ones. For example, serving adjacency queries or maintaining and querying low-cost snapshots for

archival purposes are common operations in such critical applications, and can benefit from the use of in-memory representations of graphs.

The graphs we are interested in representing share some common features. First, they represent huge networks extending to millions of nodes, but the degrees (in/out-degrees) of the latter are power law distributed [12,9], rendering the graphs to be rather *sparse*. Moreover, the graphs exhibit the *locality of reference* property: nodes tend to have successors that are ‘close’ to them in a sense that depends on the context and the nature of the network. For instance, web pages often contain links to pages of the same web site or domain, and people in social networks are often friends with individuals from the same neighborhood. Furthermore, these graphs exhibit the *copy* property (or *similarity* property), which denotes that nodes occurring close to each other tend to have many common successors.

These properties induce various types of redundancy in the graphs’ representations, and are taken into account when designing compression methods. The state-of-the-art approach to the compact representation of graphs is the method of Boldi and Vigna [5], further improved using a reordering of the graph [6] before compressing it. The reorderings can favor any compression algorithm that takes the aforementioned properties into account.

The web and social graphs may share the above properties, but feature a substantial difference in the way they are represented: while it is easy to order the nodes of a web graph in a meaningful way which favors its compression (i.e., lexicographically by URL), there is no such obvious ordering for general networks, including social ones. This makes social and general networks less amenable to compression than web graphs and their compression is a challenging issue.

*Related work.* The need for compact representations of graphs emerged with the explosion of the size of the worldwide web, so the first such attempts focused on compressing web graphs. In the last dozen years graph compression has turned into a very active research area and many algorithms have been proposed, some of them designed for more general graphs like the social network ones. Most algorithms in this direction try to offer a good space/time trade-off.

The graph compression algorithms that have been proposed so far can be classified in the following three main categories: (i) algorithms for compressing web graphs, (ii) algorithms for compressing (also) more general graphs (mostly social network graphs), and (iii) algorithms that include or employ reordering of the graph in order to favor higher degree of compression. It is also very often the case that specific web graph compression algorithms were later enriched with new techniques in order to be able to compress social graphs as well.

In [37] the authors take into account the locality of reference and the copy properties for the case of the web and initiate research on web graph compression by maintaining compressed forms of the graph’s adjacency lists. The highest compression ratios are achieved by the method of Boldi and Vigna [5], combined with a reordering using label propagation [6]. The WebGraph com-

pression method introduced in [5] is indeed the most successful member of a family of approaches [36,9,1,42] for compressing *web* graphs based on the statistical properties described in the introduction. In another line of work, Brisaboa et al. [8] propose a compact representation of the adjacency matrix that represents the graph. The approach we propose is to some extent similar to [8], in the sense that we represent parts of the adjacency matrix of a given graph. The main difference with our approach is that we represent only some *dense* parts of the graph, those that are close to the main diagonal. In [12] Chierichetti et al. view the problem of graph compression from a theoretical point of view and study the extent to which a large social network can be compressed. Their proposed method, however, is a compression scheme rather than a compressed data structure, as noted in [6], i.e., it aims solely at minimizing the size of the compressed graph (bits/edge) instead of providing fast access to each edge.

The locality of reference property of a graph reflects on its adjacency matrix in the following way: using a proper ordering of the nodes' labels, i.e., an ordering in which labels of densely connected nodes are close to each other, many edges fall close to the main diagonal of the adjacency matrix. Such orderings are preferred in practice, but finding the ordering that minimizes the distance of the edges from the main diagonal is *NP*-hard [41]. In [7] Boldi et al. test some known orderings of the nodes and propose some new ones, and study their effect on the compression of web and social graphs.

Some methods that claim to yield lower bits/edge ratios [12,10,28] do not address the issue of retrieving the edges fast. In [19] the authors introduce SLASH-BURN, an ordering method that offers the best bits/edge ratio according to the information theoretic lower bound, among other competing methods.

*Our contribution.* We concentrate on the compression of web, social network and other similar graphs by exploiting the locality of reference property. After observing that all real graphs of the above types we tested, as well as most graphs created by human activity, demonstrate the locality property, i.e., they can be represented by adjacency matrices with high concentration of edges around the main diagonal of the matrix, we exploited this fact to improve their compression.

Since the highest compression ratios are achieved by the state-of-the-art algorithm of [5] (denoted as **BV**) after applying the *Layered Label Propagation* (LLP) algorithm [6] on the input graphs, we decided to build on it, achieving the following:

- we improved **BV** by exploiting the locality property in a different way than in [5] and, thus, went beyond the state-of-the-art in graph compression
- we evaluated experimentally our method using real datasets and clearly verified that it achieves a better compression ratio than **BV**, while allowing faster retrieval of the elements of the graph than **BV**.

We assume that we apply our algorithm on a reordered version of the input graph, using for example the reordering algorithm of [6].

In our proposed method [25] we isolate the dense part of the graph’s adjacency matrix, which lies around its main diagonal (referred to as the diagonal stripe), and use a bit vector to represent it, while we resort to BV to compress the remaining edges. Every possible pair of nodes lying in the diagonal stripe is mapped through a simple function to the bit vector. Thus, the existence of an edge there can be verified in constant time. A large percentage of these pairs represent edges absent from the graph; however, including them in our representation allows us to be aware of the position of every pair and not resort to using an index as in [8], which would not only introduce a similar space overhead, but would dramatically increase the retrieval time as well. By using BV to compress the rest, sparse part, of the graph, we manage to provide a full graph compression framework and perform comparisons over the whole graph, not only the diagonal stripe. The computational complexity of this approach is approximately equal to the complexity of BV alone, as mapping the diagonal stripe to a bit vector is linear in the number of diagonal edges. Furthermore, this mapping can only decrease the query time on the compressed graph’s elements when compared with the query time of BV alone. We tested our approach on a dataset of six real social network graphs. Our method outperformed BV for the whole dataset, showing that the effect of our observation is powerful on social network graphs.

In order to make our method efficient for a wider class of graphs, including web graphs and routing networks, we refined it further [26]. Using data compression techniques that exploit the redundancy of the diagonal stripe, allows us to reduce the size of the stripe significantly. Shannon’s source coding theorem states that it is impossible to compress with an average number of bits per symbol less than the entropy of the source. We present a proposition that imposes an upper bound to that limit, and provides us with an estimation of the space requirements of our method for the dense part of the graph. Comparing this estimation for various widths of the diagonal stripe of a graph, to the compression ratio of the state-of-the-art method, we may assess the overall room for improvement and the optimal width of the stripe. However, the estimation on the latter is far from accurate due to the delicate balance between easing the task of compressing the rest of the graph by including as many edges as possible in the diagonal stripe, on one hand, and minimizing its ratio, on the other. We still wish to retain the ability to access the elements of the stripe in constant time after compressing them. Therefore, we encode them using a form of lossy, but fixed-length encoding to preserve the direct access of the edges. The edges that are excluded during this step are added to the ones existing outside the diagonal stripe. These edges will be then compressed using BV, thus, our overall method is lossless. The dataset that we used to apply and test our compression technique, comprises nine well-studied *web*, *road network*, *citation*, and *social network* graphs. Our method still outperforms BV, with impressive results for road networks (>20% decrease in compressed size), a large impact on social network graphs (approximately 16.6% decrease) and improvements even on web graphs. There are significant improvements in access times as well.

The results of this work are presented in chapter 5 of the thesis [35] and have been published in [25,26]. The developed software is open-source, publicly available at: <https://bitbucket.org/network-analysis/bvplus>

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# Computational Methods for the Identification of Statistically Significant Genes: Applications to Gene Expression Data of Various Human Diseases.

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**Abstract.** In this dissertation, we address the problem of gene selection from ranked gene lists. We propose a new hybrid feature selection method (mAP-KL) that combines successfully multiple hypothesis testing and affinity propagation clustering algorithm along with the Krzanowski & Lai cluster quality index, to select a small yet informative subset of genes. We subject our method across a variety of validation tests on simulated microarray data as well as on real microarray data. The overall evaluation results suggest that mAP-KL generates concise yet biologically relevant and informative  $n$ -gene expression signatures, which can serve as a valuable discrimination tool for diagnostic and prognostic purposes, by identifying potential disease biomarkers in a broad range of diseases. Finally, to provide the research community with the capability to apply mAP-KL in any given gene expression dataset, we have implemented this methodology to a Bioconductor/R-package accompanied with extra functionalities.

**KEYWORDS:** microarrays, gene expression data, significance analysis, hybrid feature selection, biomarkers

## 1 Introduction

The dawn of DNA microarray technology has improved our potential to comprehend the underlying mechanisms of human diseases and to aid in more accurate classification, diagnosis, and/or prognosis. Because of its high throughput nature, computational tools are essential in data analysis and mining in order to help biomedical researchers to maximize the extracted knowledge from the experimental results. In the area of diagnostics, microarray-derived markers are emerging as a valuable tool. Similar to any other clinical test, the primary goal of molecular tests, including microarray tests, is to provide reliable and timely results for improving patient care. In order to maximize the usefulness of microarrays in the diagnostic/prognostic arena it is important to

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minimize the number of biomarkers that need to be tested for an accurate diagnosis to be reached.

The selection of those biomarkers, however, is a challenging process in which feature selection (FS) methods could make a significant contribution. Indeed, from the late 90s a plethora of methods emerged and applied on several microarray studies. Despite differences in their fundamental algorithms, they all share the same objectives: 1) to avoid overfitting and improve prediction performance; 2) to make faster and cost effective models; and 3) to offer a deeper insight into the underlying processes [1]. Nevertheless, selecting those ‘significant’ genes that perform the same level of classification in relation to a specific disease is far from feasible at the moment and still an open issue.

## **2 Related work**

In reality, every microarray dataset may result to as many significant gene lists to as many FS methods we apply. Even in cases where methods share the same principles the produced gene lists are bound to diverge. Speaking of methods that share common principles, we may define the following broad groups of FS methods. Filtering, wrapper and embedded FS methods are the key categories in the field, each one with the respective advantages and disadvantages. In addition to this classification, a new class of FS methods, hybrid methods, has emerged. Hybrid methods’ combine methods of different categories aiming at taking advantage of their pros while alleviating their cons of benefit to the ‘significant’ gene list selection.

Combining methods is a constructive decision making process based always on scientific assumptions, either biological or statistical, rather than on pot luck. For instance, Jaeger et al. [2] claimed that ranking algorithms produce lists of genes, where the top ranked genes are highly correlated with each other, mainly because they belong to the same pathway. Additionally, Hall in his thesis [3] investigated the hypothesis that “A good feature subset is one that contains features highly correlated with the class, yet uncorrelated with each other”. Those beliefs were the springboard for several hybrid methods, which combined a ranking (filtering) method and a clustering method to conclude to a list of significant genes.

In particular, Jaeger et al. employed a fuzzy clustering algorithm to prefilter the genes by grouping them according to their similarity. Then, with the aid of a statistical test like t-test or Wilcoxon, selected one or more representative genes from each cluster to form a list of ‘significant’ yet uncorrelated genes. In this study, the number of clusters to be formed and the number of representative genes remained unaddressed. Similar to Jaeger et al., Hanczar et al. [4] proposed a two step method where an unsupervised clustering method, K-mean, combined with a mathematical notion, prototype gene, that tries to identify the representative genes of each cluster. Analogous issues to Jaeger et al. appeared in this study, and characterized as objectives for future work by the researchers. An alternative algorithmic approach, where ranking of genes precedes any other method is described in the mRMR [5] method. Particularly, the initial ranking through t-test or F-test is then combined with a se-

quential iteration between pairs of the ranked genes, to conclude to a subset of ‘significant’ genes according to some criteria, maximum relevance and minimum redundancy. One considerable drawback of this approach is that the redundancy criterion may exclude genes that considered important from a biological point of view. Another interesting approach, HykGene [6], proposed a three step gene selection, which incorporates a filtering algorithm, a hierarchical clustering on the top-ranked genes and finally a sweep-line algorithm that first identifies the clusters from the dendrogram and then selects one representative gene per cluster.

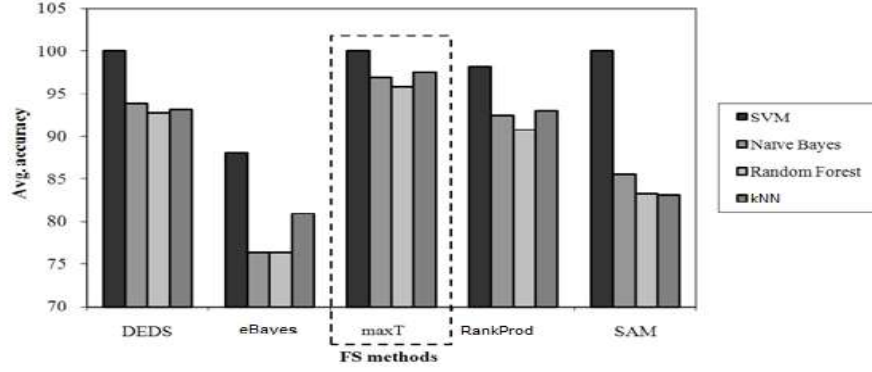
Taking into account the promising classification results of those combined methods as well as their intrinsic limitations, we considered a new hybrid method, *mAP-KL* [7]. In the proposed approach, the genes are first ranked according to their differential expression using a multiple hypothesis t-test, which controls successfully the Type I error. Then the top  $N$  ranked genes are held and grouped to clusters with the Affinity Propagation (AP) clustering algorithm [8]. Prior to AP a clustering index algorithm determines the number of clusters among the top- $N$ -genes. The output of this method is a subset of genes, one exemplar per cluster that best describes the phenotypes’ characteristics.

### 3 Proposed Hybrid Feature Selection method (*mAP-KL*)

A FS method, in microarray gene expression data, should be independent of platform, disease and dataset size. Our hypothesis is that among the statistically significant ranked genes in a gene list, there should be clusters of genes that share similar biological functions related to the investigated disease. Thus, instead of keeping  $N$  top ranked genes, it would be more appropriate to define and keep a number of gene cluster exemplars. We propose a hybrid FS method (*mAP-KL*), which combines multiple hypothesis testing and AP clustering algorithm along with the Krzanowski & Lai cluster quality index [9], to select a small yet informative subset of genes.

#### 3.1 The Filtering method

The proposed methodology combines ranking/filtering and cluster analysis to select a small set of non-redundant but still highly discriminative genes. In relation to the filtering step, we first employ the *maxT* [10] function to rank the genes of the training set and then we reserve the top  $N$  genes ( $N = 200$ ) for further exploitation. Our decision on which FS method to employ follows the findings of an analysis that we carried on FS methods [11]. Specifically, we assessed the classification performance of five different FS methods on data from ten different neuromuscular diseases. Each method yielded a different ranked list of genes, which was then used iteratively from top to bottom, in the range of 2 to 400 genes, to compose a new classification scheme in each iteration. The evaluation of the classification performance of all the produced schemes per FS method is depicted in Figure 1, and shows that the *maxT* achieved an average discrimination accuracy of 95%, between normal and disease samples.



**Fig. 1.** The overall classification accuracy of five feature selection methods on ten datasets of neuromuscular disease data according to four classification algorithms

### 3.2 The Clustering Quality index

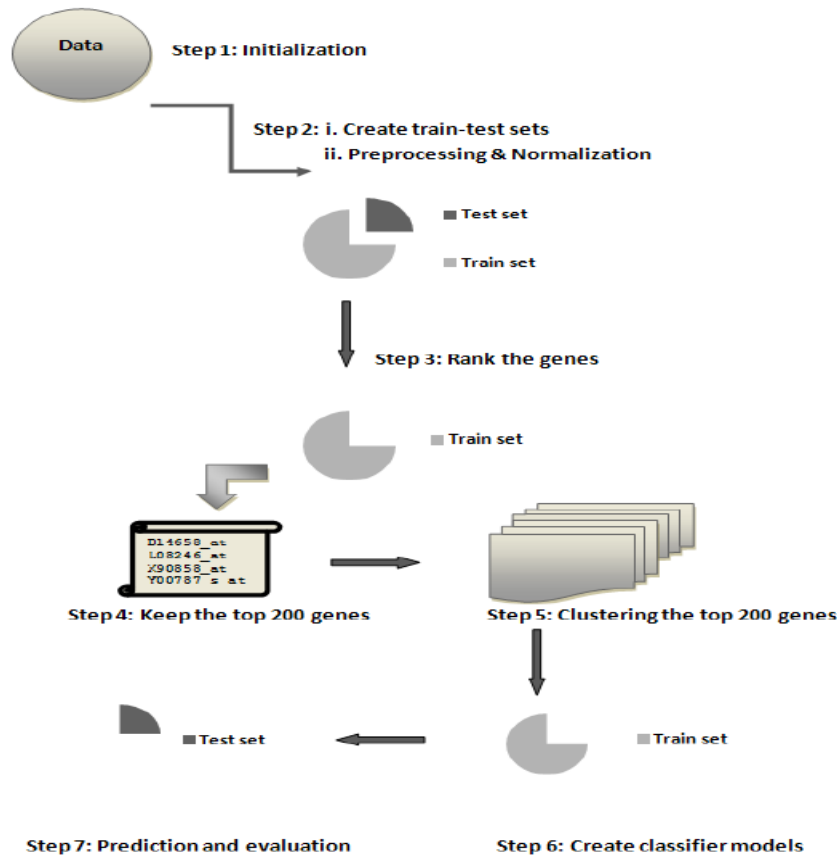
In the sequel, prior to clustering analysis we define the number of clusters, which in essence will be the number of representative genes that finally will compose our subset. The decision about which quality index to use, was based first on the indices comparison results of the Tibshirani et al. [12] survey as well as on several trials on simulated clustering data that also proved the efficiency of the index. Hence, we employed the index of Krzanowski and Lai to determine the number of clusters solely on the disease samples of the training test set.

This is actually a very fine detail in our methodology, since it has a direct impact on the clusters identification and consequently on the selected genes. However, we came across a dilemma regarding the part of the data that it would be the most proper and advantageous to apply the index. The first option was to search for the clustering structure solely in the samples belonging to the normal/control phenotype, whereas the second alternative was to investigate the samples in the disease phenotype. We finally reckoned that what actually is of interest for the identification of significant genes relevant to a disease, is the disease part of the data because all the information about the ‘triggered’ molecular processes is definitely present in it.

### 3.3 The Affinity Propagation Clustering Algorithm

The final step of our methodology involves the cluster analysis through the AP clustering method. The AP algorithm appeared in the late 20s and according to a benchmark analysis [13] across 15 other clustering algorithms, including k-means and k-medians clustering, hierarchical agglomerative clustering e.t.c., excelled at finding the more accurate clustering solution. Besides its intrinsic belief that initially all data points (genes) are considered as potential exemplars and its efficient convergence to the final clustering, urged us to adopt AP as an indispensable part of our methodology. Thus, we pass into AP the number of  $k$  clusters according to the Krzanowski and

Lai index and then let AP to detect those  $n$  clusters where  $(n = k)$  clusters among the top  $N$  genes (a pre-defined number). The algorithm converges to the requested number of clusters (most of the times) and provides us with a list of the most representative genes of each cluster, the so called exemplars. These  $n$  exemplars are expected to form a classifier that shall discriminate between the normal and disease classes in a test set. Finally, we formulate the updated train and test sets by keeping only those  $n$  genes, and proceed with the classification process. The general flowchart of our methodology appears in Figure 2.

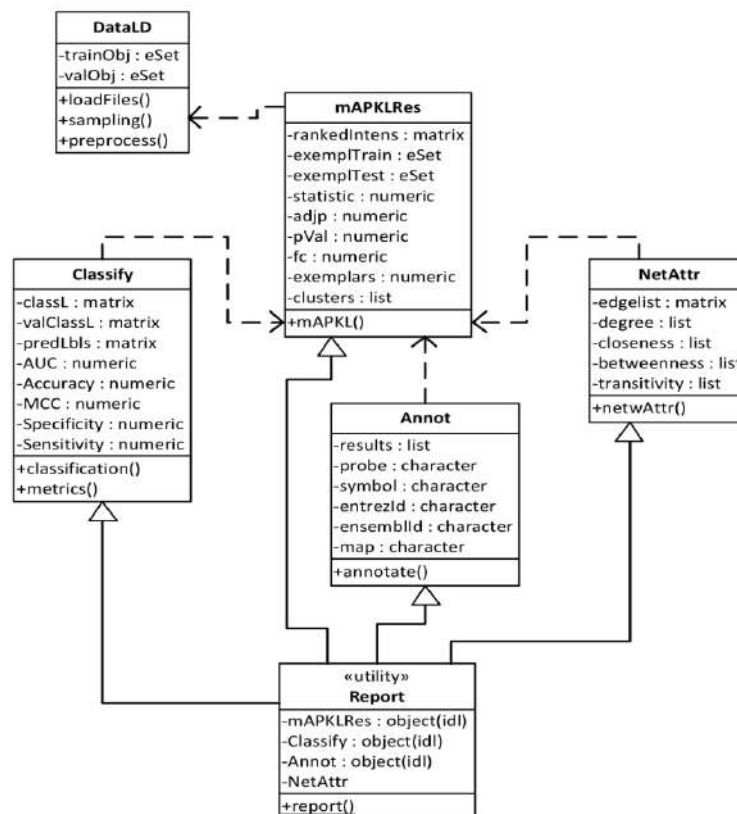


**Fig. 2.** The mAP-KL methodology flowchart

### 3.4 The Implementation of mAP-KL into an R-package

To provide the research community with the capability to apply mAP-KL in any given gene expression dataset, we have implemented this methodology to an open-source Bioconductor/ R package accompanied with extra functionalities such as data

sampling preprocessing, classification, network analysis, gene annotation analysis, pathway analysis and reporting that collaborate through five built-in classes, Figure 3. The centric idea during the package's design was to build functions that either can shape an extensive analysis pipeline or used as standalone modules. For instance, a user may import any dataset of raw gene expression data and apply with a single command eight at maximum different preprocessing methods. Then, may analyze any of the preprocessed data with the mAP-KL method and conclude to lists of significant genes (exemplars). Classification assessment, annotation analysis, pathway analysis and network characteristics are some of the possible analyses that a user may apply on these exemplars. On the other hand, a user may as well employ any of the available functions to exploit a particular functionality for example, to partition a dataset into train and validation sets, to obtain annotation info for a given list of probe ids, and so on.



**Fig. 3.** A UML schematic representation of the classes and functions of the mAPKL.

## 4 Results and Discussion

We subjected our method to a series of evaluation tests on simulated microarray data in the first part and real microarray data in the second. Regarding the real microarray data we employed datasets of six neuromuscular diseases as representatives of small cohorts and four cancer datasets with numerous samples per phenotype. We designed and executed an elaborate set of analytical experiments with 5-CV on the training set and hold-out validation on a separate set using three different classifiers, RF – SVM – KNN, to assess its performance across whole genome expression datasets from both small and large patient cohorts. Moreover, on those microarray datasets we also applied 12 other feature selection/elimination approaches and compared the classification results using several metrics, for example AUC, TNR, TPR. In particular, we employed six univariate filter methods (eBayes [14], ODP [15], maxT [10], SAM [16], SNR and t-test [17]), one multivariate filter algorithm (cat [18]), three dimension reduction approaches (BGA-COA [19], PCA [20], PLS-CV [21]), one embedded method (Random Forest [22]), and one hybrid method (HykGene [6]).

Apart from the classification analysis, we investigated the produced gene lists from a biological perspective. The power of any FS approach is evident not only from its classification performance, but also from the biological relevance to the respective pathological phenotypes. Therefore we engaged the produced gene lists from mAP-KL and the methods that excelled in the classification process, (eBayes, PLS-CV, SAM, BGA-COA, RF-MDA), as well as the maxT method which is the ranking method of mAP-KL, into a series of validations. During those validations, we tried to unravel the ‘semantics’ behind those gene lists and its association with the respective diseases.

### 4.1 Assessing the Classification Performance on Microarray Data

The overall results, based on the RF classifier, as summarized in Figure 4 places mAP-KL at the top shelf among 12 other FS algorithms developed for the mining of gene expression data. In particular, the mAP-KL method achieved the second best mean AUC in neuromuscular diseases i.e. 0.91 and the sixth best in cancer data. Eventually, the classification performance of mAP-KL across all ten diseases reached the AUC score of 0.86, which is the third best AUC score with the minimum standard deviation value compared to the methods with better classification performance e.g. eBayes, PLS-CV. Hence, we may firmly state that the combination of a univariate and a clustering method isolates subsets of genes that may discriminate unknown samples from a variety of diseases and number of samples quite accurately.

	eBayes	PLS-CV	SAM	BGA-COA	RF-MDA	mAP-KL	cat	Hyk Gene	maxT	ODP	SNR	t-test	PCA	MEAN
Diseases with Small Sample Size available	ALS	1.00	1.00	1.00	1.00	1.00	1.00	0.64	1.00	1.00	1.00	1.00	1.00	1.00
	DMD	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.61	0.97
	JDM	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	LGM2A	1.00	1.00	1.00	1.00	1.00	1.00	0.94	0.94	1.00	0.94	1.00	0.58	0.96
	LGM2B	0.48	1.00	0.52	0.98	1.00	0.70	0.36	0.82	0.91	0.73	0.88	0.82	0.72
	MM	-	0.42	0.65	0.47	0.22	0.74	0.78	0.88	0.37	0.25	0.90	0.89	0.55
	MEAN	0.90	0.90	0.86	0.91	0.87	0.91	0.86	0.88	0.87	0.83	0.95	0.95	0.66
Diseases with Large Sample Size available	BREAST	-	0.82	0.77	0.76	0.82	0.87	0.75	0.76	0.77	0.74	0.77	0.73	0.75
	COLON	0.80	0.79	0.80	0.87	0.81	0.89	0.80	0.81	0.79	0.82	0.79	0.83	0.82
	LEUKEMIA	1.00	0.99	0.99	1.00	0.99	0.71	0.99	0.97	0.96	-	0.50	0.50	0.64
	PROSTATE	0.86	0.87	0.92	0.73	0.83	0.80	-	0.69	0.50	-	0.50	0.50	0.70
	MEAN	0.89	0.87	0.87	0.84	0.86	0.82	0.85	0.81	0.76	0.78	0.64	0.63	0.68
TOTAL MEAN	0.89	0.89	0.87	0.87	0.87	0.86	0.85	0.84	0.81	0.81	0.80	0.79	0.67	
TOTAL STD	0.184	0.185	0.173	0.179	0.242	0.127	0.214	0.129	0.223	0.259	0.191	0.197	0.240	

< 50	0.50-0.69	0.70-0.79	0.80-0.89	0.90-0.95	0.96-0.99	1.00
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Fig. 4. The overall classification results (AUC metric) with RF classifier

## 4.2 Biological relevance of discriminatory gene lists

Typically, the initial product of an FS method is a list of ids rather than gene symbols, since the expression data stem from microarray chips technology. Therefore, a necessary action that we typically take is to match those probe ids with the relevant gene symbols. Another interesting thing from chip technology is that one gene symbol is regularly represented by more than one probe ids. Thus, an over or under expressed gene may be present in a top ranked list more than one times according to the chip specifications. As a result, those multiple instances of a gene shall be removed from any top ranked list to conclude to a list of unique top genes. This is an essential step regarding the anticipated gene enrichment since a top list of 20 or 50 probe ids may for example represent 14 or 35 unique gene symbols. Furthermore, gene chips include internal and external spiked in controls responsible for the hybridization quality that should be not included in the top ranking of any differential analysis. For all those reasons, the 'degree of uniqueness' (DoU) of a top ranked list is a first validation measure directly connected to the list's potential from a biological standpoint.



In the following tables, we have cited the number of probe ids and the respective number of gene symbols per method and per dataset. In the last column, we have calculated the DoU value as the average of the division between gene symbols and probe ids. The closest to the unit the more unique is the ranked list. Regarding the neuromuscular data, Table 1, the *mAP-KL* achieved the highest score with the *maxT* being quite close. In relation to cancer data, Table 2, the *eBayes* method surpassed the other methods although its average quantity is based on three rather than four datasets. The *mAP-KL* placed second setting a direct inference about the high “uniqueness” of the produced lists.

**Table 1.** The DoU of seven FS methods across neuromuscular data

FS	ALS		DMD		JDM		LGMD2A		LGMD2B		NM		DoU
	Prbs	Gns	Prbs	Gns	Prbs	Gns	Prbs	Gns	Prbs	Gns	Prbs	Gns	
<b>mAP-KL</b>	21	20	14	14	21	20	6	6	15	15	18	18	<b>0.984</b>
<b>maxT</b>	20	20	20	20	20	20	20	20	20	20	20	18	0.983
<b>RF-MDA</b>	20	20	20	20	20	20	20	19	20	20	20	18	0.975
<b>SAM</b>	20	14	20	20	20	18	20	16	20	16	20	20	0.867
<b>eBayes<sup>1</sup></b>	20	17	20	20	20	18	20	16	20	15	-	-	0.860
<b>PLS-CV</b>	20	13	20	20	20	19	20	18	20	16	20	17	0.858
<b>BGA-COA</b>	20	15	20	17	20	18	20	14	20	17	20	17	0.817

<sup>1</sup> The *eBayes* method evaluated in five datasets

**Table 2.** The DoU of seven FS methods across cancer data

FS	Breast		Colon		Leukemia		Prostate		DoU
	Prbs	Gns	Prbs	Gns	Prbs	Gns	Prbs	Gns	
<b>eBayes<sup>1</sup></b>	-	-	20	18	20	18	20	19	<b>0.917</b>
<b>mAP-KL</b>	6	4	20	16	5	5	12	12	<b>0.867</b>
<b>PLS-CV</b>	20	14	20	18	20	19	20	17	<b>0.850</b>
<b>BGA-COA</b>	20	12	20	18	20	19	20	18	<b>0.838</b>
<b>SAM</b>	20	11	20	18	20	18	20	19	<b>0.825</b>
<b>maxT</b>	20	11	20	16	20	17	20	20	<b>0.800</b>
<b>RF-MDA</b>	20	9	20	14	20	18	20	19	<b>0.750</b>

<sup>1</sup> The *eBayes* method evaluated in three datasets

A second validation criterion is the enrichment of the unique gene symbols in relation to the associated pathways. At this point is crucial to refer to another parameter before mentioning the results of this validation measure, which are the protein-coding-genes (P-C-Gns) in the ranked list. In essence, not all of the known genes are protein coding and thus involved in molecular functions. Pathway analysis tries to simplify the complexity at the cellular level through the representation of a series of

steps where “each step is an event that transforms input physical entities into output entities” [23]. Such entities are definitely the produced proteins, among other small molecules or particles, and as a consequence only the protein coding genes are requisite for a pathway analysis.

Through a plethora of pathway analysis tools, we utilized the ‘Reactome’ pathway database [23], which is a curated and peer reviewed database of pathways and reactions in human biology. We uploaded the top lists of the selected FS methods for all diseases and evaluated their pathway enrichment. During the pathway evaluation, we took into consideration the DoU and the number of protein-coding genes parameters as well as the number of pathways according to the ‘Reactome’ database. The final pathway enrichment (PE) score for each FS (m) is the average of the summation of pathways per protein-coding genes multiplied by the DoU for all diseases (d)

$$PE_m = \sum_{d=1}^{10} \frac{Protein - coding - genes_d}{Pathways_d} \times DoU.$$

We summarized the results, Table 3, where the FS methods are in descending order based on their average PE score. In accordance with the pathway analysis, the maxT method appears to achieve the highest PE score across all diseases. Besides is the method with the second highest DoU score marginally behind mAP-KL. However, this significant advantage over mAP-KL and RF-MDA that follow is mainly due to the weird PE score in prostate cancer (4.33), where the maxT identified three (3) pathways with 13 unique genes. Albeit, those three methods appear to constitute a group with PE scores close to unit, which is a satisfactory if not intriguing case for biologists.

**Table 3.** The overall pathway analysis results

FS	Pathway Analysis										Mean	Stddev
	ALS	DMD	JDM	LGMD2A	LGMD2B	NM	Breast	Colon	Leukemia	Prostate		
maxT	1.00	1.08	1.08	0.43	1.36	1.01	0.47	0.80	0.79	4.33	1.24	1.12
mAP-KL	1.43	0.78	1.38	0.43	0.88	1.40	0.67	0.63	0.80	1.17	0.95	0.36
RF-MDA	0.75	1.10	1.40	0.74	0.63	1.80	0.54	0.63	0.80	1.03	0.94	0.40
eBayes <sup>1</sup>	0.37	1.50	0.90	0.64	0.67	-	-	1.08	1.26	0.86	0.91	0.36
PLS-CV	0.37	0.89	1.21	0.66	0.90	0.85	0.98	0.90	1.07	1.04	0.89	0.23
SAM	0.29	1.13	1.00	0.64	0.80	1.08	0.46	1.15	0.98	1.27	0.88	0.32
BGA-COA	0.68	1.06	0.63	0.70	1.19	0.85	0.60	0.90	1.14	1.00	0.87	0.22

<sup>1</sup> The eBayes method evaluated in eight datasets

## 5 Conclusions

We proposed a hybrid FS method (mAP-KL), which clearly demonstrates how effective the combination of a multiple hypothesis testing approach with a clustering algo-

rithm can be to select small yet informative subsets of genes in binary classification problems. Particularly, across a variety of diseases and datasets, mAP-KL achieved competitive classification results compared to other FS methods and specifically to HykGene method, which follows a similar philosophy i.e. first ranking and then clustering. The advances of mAP-KL over HykGene or other similar approaches stem from three key characteristics; the data-driven nature, the affinity propagation clustering, and the classifier independence. Indeed, the engagement of a cluster quality index, the Krzanowski and Lai, diminishes any fuzziness and provides the clustering algorithm with a representative number of potential clusters. Moreover, in mAP-KL the data determine the size of the subset i.e. the structure of the data dictate the number of clusters and the clustering algorithm decides on the representatives upon each cluster. Contrary to other methods, for example HykGene, where a classifier is wrapped around its method, in our case no classifier takes part during the subset construction. This methodological characteristic is of great importance since our subsets lack of any overfitting phenomenon pertinent to classifiers.

Relevant to the identification of clusters, the employment of AP clustering algorithm, deals effectively with the issue of representative genes per cluster. Other comparable approaches to mAP-KL admitted considerably difficulties on selecting effectively one or more representative genes per cluster. Besides, the AP follows a gene-network mechanism by considering initially all genes as nodes in a network. The resultant exemplars are the central genes within a cluster of genes and probably the key nodes within a network of genes. Therefore mining the exemplars can be considered as the forefront of a network inference process rather than just the outcome of a FS approach. As such, we intent to construct networks based on the top  $N$  genes of our methodology and then to exploit the network characteristics of the exemplars. An initial attempt towards this direction is already available in the mAPKL package, though more network inference methods for the reconstruction of gene regulatory networks and methods for functional enrichment will be engaged in the near future.

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# Learning Enhanced Situation Perception for Self-Managed Networks

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**Abstract.** The networks in the future it is envisaged that they will be able to operate in an autonomous manner. At the same time, the networks will be able to adapt their operation for handling new challenges. This thesis aims at providing a scheme for situation aware networking, based on a hierarchical architecture, which enables the network elements to operate in a self managed way taking into consideration the local and the greater (domain) view. In this thesis we propose a scheme based on fuzzy logic for the identification of optimization opportunities or problems, a functionality called situation perception. The proposed scheme has been applied and evaluated in three networking problems (i.e., identification of QoS degradation events<sup>1</sup> for VoIP, Load events<sup>2</sup> identification for WiFi APs, and Cooperative power control). Additionally, for handling the need for reconfiguration, in case of changes in the environment, we propose the enhancement of the situation perception mechanisms with two learning schemes (a supervised one and an unsupervised one). The enhancement is related to the adaptation of the environment modeling of the fuzzy reasoners. The proposed solution is described in a generic way so as to enable its application in other problems that have similar characteristics.

**Keywords:** autonomic networking, situation awareness, situation perception, fuzzy logic, supervised learning, unsupervised learning

## 1 Dissertation Summary

Moving towards future networks, the predictions indicate that mobile and wireless data traffic will increase considerably. Mobile data traffic will increase

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globally 13-fold between 2012 and 2017 whereas global IP traffic has increased more than fourfold in the past five years [1]. Additionally, a huge increase will take place regarding the number of the connected devices (100 billions). On the other hand, future networks will be more dynamic and complex compared to the contemporary ones, including new technologies, new services, and new demands from the users, new business cases. Thus, the network elements will have to be agile and dynamic so as to manage the users' networking needs and the network operational environment. In such a complex environment, where billions of diverse devices will (simultaneously) ask for resources from the network, networks' self-management aroused as a potential solution.

Additionally, we should consider that the future network conditions are hard to predict. This implies that the networks should be able to configure their operation according to the new network stimuli, so as to handle unpredicted problems, optimization opportunities, and network conditions. For facilitating a network element to operate in the future demanding networks, a network element shall have the ability to monitor its environment, identify its state, make decisions, make projections, execute, and learn based on the previous actions. The literature analysis highlights an attempt of the self-aware schemes to mimic the human reasoning. However, due to their static definition of the environment, the schemes fail to meet requirement for human behavior approximation. Additionally, up to now both academia and industry attempt to handle the problem of environment modelling using rules and policies, combined with thresholds. Thus, we observe that a major gap of sophisticated solutions in the available proposals both in the literature and the industry solutions exists (i.e., mainly threshold based). In the following sections we present mechanisms developed in terms of this thesis for situation perception and awareness in future networks.

### **1.1 Fuzzy logic based situation perception**

The term "Situation Perception" is used to describe all correlations that take place in order to analyze data received by monitoring points and thus identify problems and select appropriate configuration actions [2] [3]. This task is considered as a complex one due to its multi-variable nature since multiple optimization goals or faults may arise, the fact that contradictory inputs may occur, and since cases with missing data that may arise. These aspects may be handled using the Fuzzy Logic algorithmic tool (i.e., fuzzy sets enhanced with rules and policies). The Fuzzy Logic based situation perception, takes into account a set of metrics/parameters, and after their joint correlation analysis,

maps them to a degree that depicts how the network elements perceives its environment (e.g., load status) [2].

A Fuzzy Logic Controller (FLC) consists of three parts, namely the fuzzifier, the inference system and the defuzzifier. The fuzzifier undertakes the transformation (fuzzification), of the input values (crisp values) to the degree that these values belong to a specific state (e.g., low, high). Then, the inference system correlates the inputs and the outputs using simple "IF...THEN..." rules. Each rule results to a certain degree for belonging to a specific state for every output. Thereinafter, the output degrees for all the rules of the inference phase are being aggregated. The actual output of the decision making process, comes from the defuzzification procedure, which captures the degree of the state of the decision maker (e.g., the network element is x% loaded; the radio link is y% interference, the user experiences z% QoS etc.). The degree is obtained using several defuzzification methods; the most popular is the centroid calculation, which returns the center of gravity of the degrees of the aggregated outputs.

## **1.2 Learning enhanced Fuzzy Logic Based Situation Perception**

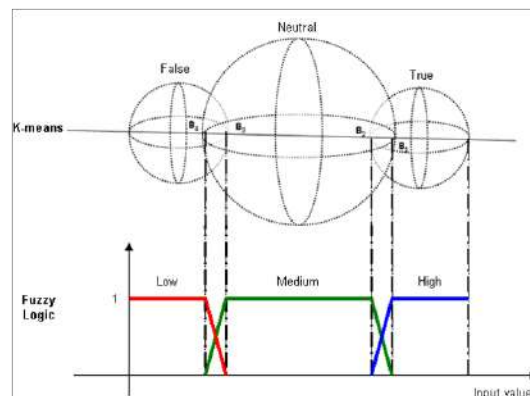
As mentioned afore, the Situation Perception schemes aim to identify problematic situations or optimization opportunities and in general perform well in the environments where they are built to operate. On the other hand, if the network conditions change/evolve, or the network elements get re-located in a totally new environment, they do not manage perform their situation perception task in a satisfactory manner, without being manually configured by the network administrators. Fuzzy logic enhanced situation perception faces the same problems regarding their adaptability as the rest of the situation perception schemes in the literature that lack inherent adaptability capabilities. In terms of this thesis two learning approach schemes are being proposed, one based on supervised learning and one on unsupervised learning. For the unsupervised learning scheme several variations of the solution have been proposed, depending on effect they have to the fuzzy logic reasoners.

### **1.2.1 Supervised Learning Algorithm**

The supervised learning algorithm is a decentralized one, with parts of the algorithm being implemented locally in each network element and others being implemented in centralized controllers. Initially, each network element monitors its environment for identifying problematic situations using the Situation Perception fuzzy reasoners. In the case that problematic situations are being identified the network element proceeds in problem solving decisions and the corresponding execution of such decisions. Afterwards, the learning

procedure takes place, which consists of three distinct phases, namely the labeling phase, the classification step, and, the fuzzy logic enhancement procedure [4] [5].

Each time a network device monitors its operational environment it extracts a d-dimensional vector which can be classified as True, indicating that a particular problem has appeared, False – no problem- or Medium/Neutral, implying that although there is currently no problem there is a chance that a problematic situation may appear in the future. Additionally, given a problem, the device triggers a remedy action, which is guaranteed to solve the problem; in other words it will enable the device to transit from a True state to either a False or a Neutral state. The correctly labelled observations are being used for the labelling of the rest of the observations, using kNN algorithm resulting in three sets, labelled as True, Neutral, and False, including all the gathered inputs, which may include misclassified observations. Thus a further step is required for ensuring that the misclassified tuples will be classified correctly. This is performed using k-Means, which is applied on the two spheres, corresponding to False and True and direct the algorithm to split it into two clusters, False or True and Neutral. The result will be two adjacent spheres maintaining elements belonging to both classes. The geometric representation of this approach is depicted in Fig. 1. The algorithm simply augments the sphere corresponding to Neutral cases and shrinks the other two by extracting falsely classified points. The intersecting points of the line defined by the spheres' centers with the spheres correspond to the desired solution, which straightforwardly leads to the identification of the input membership functions of the fuzzy logic controllers, as shown in bottom part of Fig. 1.



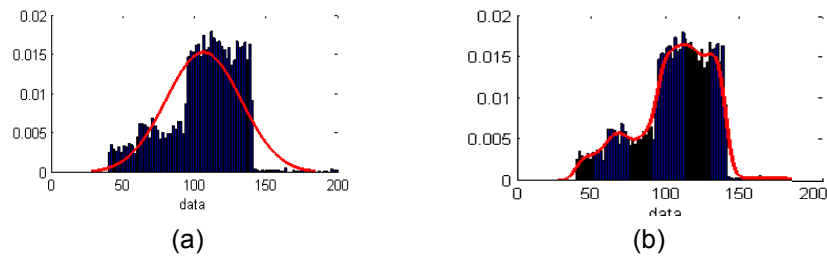
**Fig. 1:** Geometric interpretation of the approach



### 1.2.2 Unsupervised Learning Algorithm

The unsupervised version of the learning enhanced fuzzy logic situation perception scheme is based on the modification/adaptation of the previously described solution, following the same generic principles. The key difference of the unsupervised scheme lies at the skip of the complicated algorithm for labeling, following the assumption that the network administrator will have, in general, configured the network elements to operate adequately; thus enabling the system to converge. The previously described scheme exploits the measurements for the extraction of the new membership functions in a rather simple manner, using a well-known clustering method, the k-Means. This however leads to loss of information in the overlap areas of the clusters because the density of the measurements is not being considered, but only the radius of the hyperspheres is being exploited. In contrast, in the unsupervised learning scheme, the diversity of the input measurements via statistical analysis of the monitored instances is being considered [6].

Once we have gathered enough (classified) measurements we have three sets of tuples, labeled as Low, Medium and High; misclassified tuples of the diagnosis mechanism from the true negatives and false positives are also included in the three sets. The classification is based solely on the current understanding of the decision maker on what constitutes Low, Medium and High respectively. The approaches that we have followed regarding the statistical processing of the measurements are the use of the Gaussian distribution and, the non-parametric one (i.e., which uses the Kernel Density Estimator (Gaussian Kernel is used) of the measurements histogram). The former approach is simpler whereas the latter provides a better “fitting” to the available data. The mapping of both mechanisms to input membership functions is straightforward and is depicted in Fig. 2.



**Fig. 2:** (a) Mapping of a cluster to Membership functions using Gaussian statistical analysis, (b) Mapping of a cluster to Membership functions Non-parametric statistical analysis

## 2 Results and Discussion

The previously described proposals, for using fuzzy logic for situation perception problems, as well as the supervised and unsupervised learning enhanced fuzzy logic schemes have been applied in three case studies, namely QoS Degradation Events' Identification, Load events' identification, Environment modeling for Cooperative Power Control. The aim is to evaluate the mechanisms' efficiency for identifying appropriately events, as well as for adapting the model of the environment. In this section, the results of the application of the developed schemes are being briefly provided and analyzed.

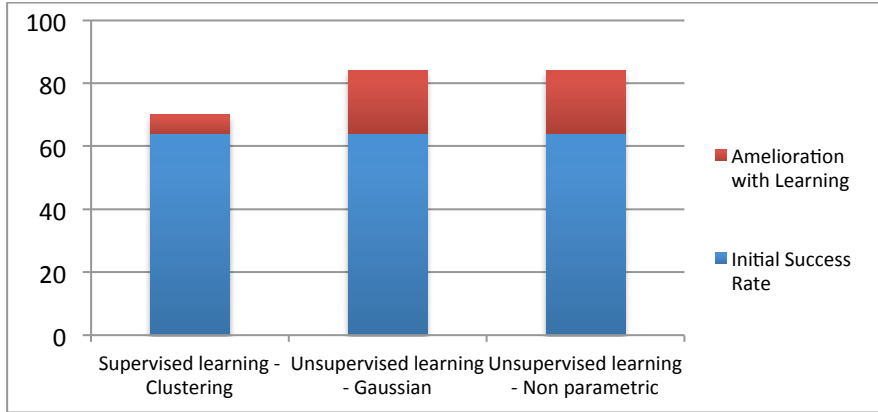
### 2.1 QoS Degradation Events' Identification

For the QoS Degradation Events' Identification and for the VoIP service, delay, jitter and packet loss are identified as playing significant role in QoS degradation, thus the situation awareness engine is based on the aforementioned inputs for every active session; for other services (e.g., highly reliable communications, IPTV etc.) other monitoring inputs could be used. The "Delay", the "Jitter" and the "Packet Loss" per flow comprise the input vector, whereas the output is the "QoS degradation". The membership functions indicate the values of each parameter, the range of each value and the magnitude of their participation. The shapes chosen for the representation of the degree of certainty are trapezoidal in the case of "Packet Loss", mainly for simplicity reasons and to so as to exploit the certainty areas for such inputs and triangular for the jitter and the delay for highlighting the symmetry and the absence of total certainty areas. For the QoS level the Gaussian membership functions are being used. The idea behind such adoption is mainly based on the smooth (i.e., the QoS should be related to the inputs in a smooth manner without non-linear alterations) and non-zero (the decision maker needs to conclude to a decision based on all inputs' range) nature at all points; for simplicity reason symmetric membership functions are being used [6].

The evaluation of the fuzzy logic based situation awareness scheme for QoS degradation events is based on 50000 tuples that have been generated randomly, using typical values available in the literature [7]. For the analysis of the dataset, and given the huge number of the considered values, the dataset is being automatically evaluated against a set of predefined fuzzy logic rules, which are strict for the identified service. The extracted labels from now on will be called ground truth and are extracted using rules that perfectly fit to the considered dataset and are only used for the evaluation of the generic definition. Using the afore described initial configuration, the success rate is 64%, which represents the number that the situation awareness scheme concluded in the same decision compared to the ground truth.

In order to quantify the benefits from the introduction of the proposed learning schemes we have conducted a series of MATLAB simulations for the evaluation of the learning enhanced situation perception. Both algorithms have been applied for evaluating them and for identifying the benefits from their introduction. By applying the supervised learning algorithm (Section 1.2.1) the situation perception algorithm has a success rate of 70.01% (amelioration of

9.4%). By incorporating the unsupervised learning mechanism with the Gaussian adaptation approach and following the methodology presented in Section 1.2.2 we modify the input membership functions accordingly. As it is obvious, the input states are now being captured by new membership functions, which are being described by Gaussian distributions, with higher overlap areas. The success rate of the adapted scheme reaches 84.07% compared to the ground truth (an amelioration of 31.36%). The required time for the processing of the dataset and the extraction of the new membership functions is 13.07 seconds in an average consumer laptop (i.e., Quad core, 1.6 GHz, 4GB RAM). For the same dataset, we also apply the unsupervised learning non-parametric approach. The new membership functions are closer to the actual distribution of the dataset, and lead in reaching a success rate of 84.16% compared to the ground truth (an amelioration of 31.51%). In this case the required time for the adaptation procedure (processing and extraction of the new membership functions) is 22.38 seconds. The results of the analysis for the three learning schemes are being summarized in Fig. 3.



**Fig. 3:** Comparative analysis of the amelioration for the three learning schemes

## 2.2 Load Events Identification

The problem of load identification is under the coverage and capacity optimization umbrella. The analysis could be extended for any inference process that a network element should execute. In this use case, each AP monitors its operational environment (Packet Error Rate, Channel Utilization, and Number of Associated Terminals) and attempts to identify potential (high) load situations [4] [5]. If such a problem occurs (high load) then they collaborate in order to select the most appropriate configuration action, which in this case is the optimal reallocation of the associated terminals among the available homogeneous or heterogeneous access points in the corresponding network area; the UEs reallocation has been extensively studied in the literature and is out of the scope of this analysis.

The shape of the membership functions (MF) is related to their special characteristics. More specifically, for the AT the MFs are trapezoidal. The key

characteristic of this MF is its simplicity and is mainly used to describe inputs that have a homogeneity degree and linear behaviour. Similarly, for the strict nature of the PER and its relation to the QoS we have decided to use the trapezoidal MFs, which describe in a satisfactory manner the considered error ranges for ideal (“low”), acceptable (“medium”) and non acceptable (“high”). Finally, the triangular MFs have been selected for the “Channel Utilization” parameter due to the linear affect of this input to a WiFi AP. In order to test the effectiveness of the proposed solution we have used three initial configurations of the fuzzy logic decision-making controller so as to capture more generic and more targeted configurations of the network equipment.

The evaluation of the proposed scheme is based on an experimental analysis for generating an evaluation dataset. This testbed was employed in order to extract the dataset used in the experimental assessment. We collected 50.000 tuples; each tuple was described by three variables indicating the status of an access point at time  $t_x$ , namely Packet Error Rate (PER) ranging in  $[0...1]$ , Channel Utilization (CU) ranging in  $[0...1]$  and Number of Associated Terminals (AT) in  $[0...25]$  [8]. 10% of the dataset has been sampled from the deployment of the testbed and has been manually labeled, while the rest has been artificially generated.

The basis of the analysis is a pre-evaluation of the dataset, with a very strict set of rules, directly capturing the environment where the APs are placed. The evaluated dataset is called ground truth and is used only for evaluation purposes. Evaluated against this dataset, all the three fuzzy logic controllers’ configurations performed well, considering ofcourse the fact that they have been generally configured. **Table 1** presents the success rate (i.e., the correctly classified tuples) using the three different configurations. We observe that the more generic a configuration is, the lower success rate he achieves, which is understandable due to the fact that the configurations matches several environments.

**Table 1:** Classification success rate results for the three configurations of the fuzzy reasoners

Configuration	Fuzzy Logic 1	Fuzzy Logic 2	Fuzzy Logic 3
Classification Success Rate	65.64%	71.86%	75.40%

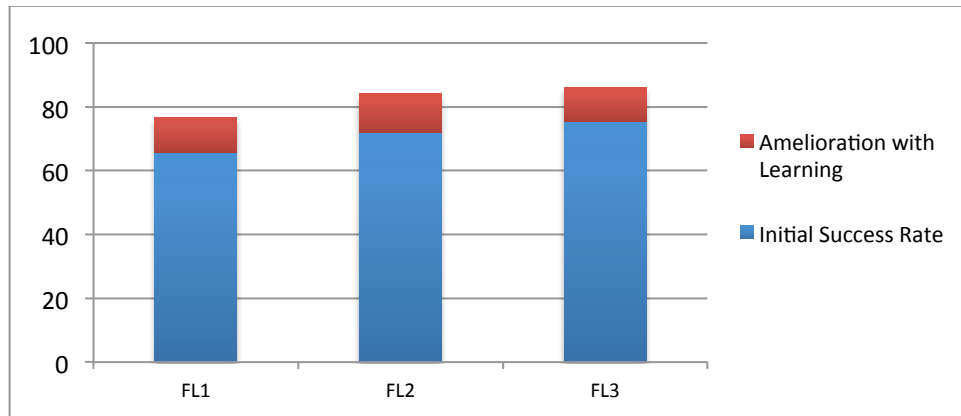
For the load events’ identification case, the supervised learning scheme has been applied for the identification of load events. For identifying the optimum value of neighbors for the kNN algorithm, we have performed several experiments with values of k 1, 5, and 10. The results have been assessed through a 10-fold cross validation procedure, while all experiments verified our initial intuition regarding the applicability of k-NN in the context of our problem exhibiting, a classification rate larger than 98%. The latter lead us to the additional conclusion that any value between 1 and 10 will provide results of adequate quality.

Based on the experiments we conclude that the algorithm performs significantly well and tends to provide environment modelling, which converge to

decisions closer to the ground truth, independently of the initial configuration of the network element's decision making engine. For the three initial configurations of the fuzzy logic controller the achieved amelioration is of 14 - 17% (**Table 2** - Fig. 4). The presented amelioration focuses on the situation awareness of a cognitive manager. The characterization of events, which is a situation awareness phase, is the pilot for the successful optimization or fix of the network system. If the cognitive manager cannot assess effectively the local status, then the performance of the network in many cases will not be improved by applying a reconfiguration action. Thus, the correct labeling of events is an important task for autonomic network management systems, where the learning process has merit.

**Table 2:** Classification results after the enhancement of the fuzzy logic rules

	Fuzzy Logic 1	Fuzzy Logic 2	Fuzzy Logic 3
Initial	65.64%	71.86%	75.40%
Learning	76.73%	84.09%	86.06%
Amelioration	16.8%	17.01%	14.13%



**Fig. 4:** Comparative analysis of the amelioration for the three initial configurations

### 2.3 Fuzzy Logic-based Cooperative Power Control

This section aims at presenting a Situation Perception mechanism for WiFi APs operating in an Ultra Dense Environment, where uncertainties may occur. The idea is to extend algorithms for cooperative power control coming from sensor networks' application field [9] [10], apply the solution in WiFi APs, and address the situation perception problem due to uncertainties that may occur in the network.

Both of the algorithms presented in [9] and [10] are based on a tradeoff between the capacity of a node and the interference caused to the correspond-

ing neighborhood. This balance is being captured by an objective function of the following type [9]:

$$A + \alpha B \quad (1)$$

The first part indicates a relation to the Shannon capacity for the corresponding user, while the second part captures the negative impact in terms of interference prices that a user causes to its neighborhood. The  $\alpha$  factor is introduced so as to capture uncertainties in the network; these uncertainties reflect the precision of the received and compiled information of each network element regarding the interference price, which should have been available by the node's neighbors. This is related to the fact that once a network element adjusts its transmission power, it informs its neighbors in an ad-hoc manner. This implies that even though a network element has collected information from all of its neighbors in order to adjust its transmission, the gathered data could be obsolete and, as a consequence, they will not capture neighborhood's current state. In [9],  $\alpha$  is set in a static manner as 25%. In [10], a fuzzy reasoner is introduced in order to identify, in a more dynamic way, uncertainties in the network based on the network's status; the inputs (number of users, mobility, update interval) of the fuzzy reasoner capture the volatile nature of the ad-hoc network, whereas the output of the fuzzy reasoner is the Interference Weight. The  $\alpha$  factor is defined as  $1/\beta$  Interference Weight + 1 ( $\beta$  has the maximum value of the Interference Weight). In our proposed scheme [11] [12] [13], we suggest using fuzzy logic controllers for the calculation of the  $\alpha$ , which afterwards shall be adapted, based on the introduction of the learning schemes.

The evaluation of the fuzzy logic enhanced scheme is based on a full day experiment [12]. In the experiment four Soekris APs have been operating enhanced with the developed solution; the transmission power of their WiFi cards is being measured throughout the experiment. The transmission power of the WiFi APs ranges from 10 to 27 dBm. For each of the Soekris devices (and considering that the 10dBm is the basis of the TxPower for each AP) we have measured the actual gain compared to setting the transmission power to the maximum TxPower (i.e., 27 dBm). The energy gain at each of APs is 12.51%, 10.75%, 33.33% and 21.23%. Also, the analysis showed that the more the APs, the more energy gains we have, due to the collaborative nature of the algorithm. Also, what should be noticed is the fact that the APs change very often their TxPower levels. This is related to the highly volatile office environment, with moving users and the many interference sources (i.e., moving users, cell phones, Bluetooth devices, etc.), in relation to the fact that the APs identify the network topology considering indoor path loss models. Such models, if we assume static environments, without moving users operate with accuracy, however in the case under discussion, the network elements need to calculate the topology on a constant basis, in every CPC loop. Throughout the experiment the SINR has been being measured and it is better compared to the case where maximum TxPower has been set to the APs. Additionally, we have been measuring the number of iterations required for the system to converge, every time the CPC is being triggered (every 5 minutes). The Soekris APs exchange messages asynchronously; everyone using its own intervals.

We observe that the scheme converges in small number of iterations most of the times (mean value of iterations 3.876).

A similar analysis has been performed when we integrated the supervised learning scheme [12]. The initial configuration of the network elements is a generic one and captures a great variety of environments. The CPC scheme is more sensitive to the environment, compared to the second day of experimentations (CPC without learning), due to the increased number of fluctuations in the TxPower setting. Given the fact that they operate in the same environment, the APs proceed even more often in transmission power adjustments. Furthermore, we observe significant energy gains, in relation to the case without learning capabilities (24.73%, 18.01%, 14.69%, 5.65% energy gains). Regarding the SINR, it remains in the same levels as in the case of the core CPC algorithm due to the fact that the objective function to be optimized is the same. The number of iterations every time the CPC is being triggered after the learning procedure slightly decreases (3.47), which also highlights that the system has become more suitable to its environment.

### **3 Conclusions**

The objective of this thesis is to analyze the concepts of the Situation Awareness and Situation Perception and present solutions for these research areas. Situation Awareness is the ability of the network elements to model their environment, assess it and interpret it so as to predict the near future. As situation perception we define the proper perception of the operational status of the system or the network element and is the primarily interpretation of the available information. In the context of this thesis, the focus has been placed on the analysis of the previous notions, and on the development of an architectural solution that enables network elements to perceive their environment correctly and efficiently. Additionally, new schemes for efficient and effective situation perception based on fuzzy logic have been proposed. These schemes have been enhanced by adaptation-learning mechanisms, so as to be able to adapt their contextual models, based on the environment stimuli. The evaluation of the proposed schemes has proven that fuzzy logic schemes for Situation Perception perform well for the environment modelling. Additionally, the developed learning schemes adapt the environment modelling and benefit significantly the performance of the Situation Perception schemes.

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